Estimating Social Effects with Randomized and Observational Network Data – Supplemental Materials –

TszKin Julian Chan
Bates White Economic Consulting
Juan Estrada
Analysis Group Economic Consulting
Kim Huynh[‖]
Currency Department, Bank of Canada
David Jacho-Chávez
Department of Economics, Emory University
Chungsang Tom Lam
Department of Finance, Florida State University
Leonardo Sánchez-Aragón
Facultad de Ciencias Sociales y Humanísticas, ESPOL University

October 3, 2024

Appendix A Primitive Conditions for Relevance

This section provides a set of primitive conditions that imply the relevance condition in Assumption 3. This is formally proven in Proposition 1 below.

Assumption A.9 $\mathbf{X}$ *is full column rank for any realization of the matrix of regressors* $\mathbf{X} \in \mathscr{X}$ *with positive probability in* $f_{\mathbf{X}_N,\mathcal{G}_N,\mathcal{G}_{N,0},\boldsymbol{\varepsilon}_N}(\mathbf{X},\mathbf{g},\mathbf{g}_0,\boldsymbol{\varepsilon})$.

Assumption A.10 *For any two regressors k and ℓ and a number $p \geq 2$, the expectation* $\mathbb{E}[\mathbf{w}^p_{N,0;i}\mathbf{x}_{N;k}\mathbf{w}_{N;i}\mathbf{x}_{N;\ell}]$ *exists for all i.*

[‖]The views expressed in this article are those of the authors. No responsibility for them should be attributed to the Bank of Canada. All remaining errors are the responsibility of the authors.

Assumption A.11 *There exists two* different *numbers* $(r, s) \in \mathbb{N}_+ \times \mathbb{N}_+$ *such that* $\mathbf{I}_N$, $\mathbf{W}^r$ *and* $\mathbf{W}^s$ *are linearly independent for any realization of the network of interest* $\mathbf{g} \in \mathscr{G}$ *with a positive probability in* $f_{\mathbf{X}_N,\mathcal{G}_N,\mathcal{G}_{N,0},\boldsymbol{\varepsilon}_N}(\mathbf{X}, \mathbf{g}, \mathbf{g}_0, \boldsymbol{\varepsilon})$. *Furthermore,* $(\gamma_{0,k}\beta_0 + \delta_{0,k}) \neq 0$ *for all* $k \in \{1, \dots, K\}$.

Assumption A.12 *The equation* $x_{i,k} \neq \mathbf{w}_i^m \mathbf{x}_k$ *holds for some number* $m \in \mathbb{N}_+$, *for all individuals* i, *any regressor* k, *and any realization of the matrix of regressors* $\mathbf{X} \in \mathscr{X}$ *and the network of interest* $\mathbf{g} \in \mathscr{G}$ *with positive probability in* $f_{\mathbf{X}_N,\mathcal{G}_N,\mathcal{G}_{N,0},\boldsymbol{\varepsilon}_N}(\mathbf{X}, \mathbf{g}, \mathbf{g}_0, \boldsymbol{\varepsilon})$.

Assumption A.13 *There exist a number* $p \geq 2$ *such that* $\mathbf{I}_N$, $\mathbf{W}_0$, $\mathbf{W}_0^2$, $\dots$, $\mathbf{W}_0^p$ *are linearly independent for any realization of the exogenous network* $\mathbf{g}_0 \in \mathscr{G}_0$ *with positive probability in* $f_{\mathbf{X}_N,\mathcal{G}_N,\mathcal{G}_{N,0},\boldsymbol{\varepsilon}_N}(\mathbf{X}, \mathbf{g}, \mathbf{g}_0, \boldsymbol{\varepsilon})$.

Assumption A.14 *The joint probability distribution* $\Pr(\mathcal{G}_N = \mathbf{g}, \mathcal{G}_{N,0} = \mathbf{g}_0)$ *is such that* $\mathbb{E}[\mathbf{w}_{N,0;i}^p \mathbf{x}_\ell \mathbf{w}_i \mathbf{x}_{N,k}] \neq 0$ *for at least* $p = 2$, *and any realizations of* $\mathbf{x}_\ell$ *and* $\mathbf{x}_k$ *with positive probability in* $f_{\mathbf{X}_N,\mathcal{G}_N,\mathcal{G}_{N,0},\boldsymbol{\varepsilon}_N}(\mathbf{X}, \mathbf{g}, \mathbf{g}_0, \boldsymbol{\varepsilon})$ *and all* i.

Assumption A.15 $\mathbb{E}[\sum_{r=0}^{\infty} \beta_0^r \mathbf{w}_{N;i} \mathbf{W}_N^r \varepsilon_N \mid \mathbf{W}_{N,0}, \mathbf{X}_{N,k}] = 0$ *for any* $k \in \{1, \dots, K\}$.

Proposition 1 *Let Assumptions A.9, A.10, A.11, A.13, A.12, A.14 and A.15 hold. It follows that the matrix* $\mathbb{E}[N^{-1} \sum_{i \in \mathcal{I}_N} \mathbf{z}_{N;i} \mathbf{d}_{N;i}^\top]$ *has full column rank.*

Proof. Let $\mathbf{w}_{N;i}$ and $\mathbf{w}_{N,0;i}$ be the ith row of the adjacency matrices representing the population network of interest and the exogenous network. Similarly, let $\mathbf{x}_{N,k}$ be the $N \times 1$ vector of the regressor k for all N individuals in the population and let $x_{N,k;i}$ be the value of the regressor k for the individual i. Therefore, it follows that for the individual i, $\mathbf{z}_{N;i} \mathbf{d}_{N;i}^\top$ equals

$$\begin{bmatrix}
\mathbf{w}_{N,0;i}^p \mathbf{x}_{N,1} \mathbf{w}_{N;i} \mathbf{y}_N & \mathbf{w}_{N,0;i}^p \mathbf{x}_{N,1} \mathbf{w}_{N;i} \mathbf{x}_{N,1} & \dots & \mathbf{w}_{N,0;i}^p \mathbf{x}_{N,1} \mathbf{w}_{N;i} \mathbf{x}_{N,K} & \dots & \mathbf{w}_{N,0;i}^p \mathbf{x}_{N,1} x_{N,K;i} \\
\mathbf{w}_{N,0;i}^p \mathbf{x}_{N,2} \mathbf{w}_{N;i} \mathbf{y}_N & \mathbf{w}_{N,0;i}^p \mathbf{x}_{N,2} \mathbf{w}_{N;i} \mathbf{x}_{N,1} & \dots & \mathbf{w}_{N,0;i}^p \mathbf{x}_{N,2} \mathbf{w}_{N;i} \mathbf{x}_{N,K} & \dots & \mathbf{w}_{N,0;i}^p \mathbf{x}_{N,2} x_{N,K;i} \\
\vdots & \vdots & & \vdots & & \vdots \\
\mathbf{w}_{N,0;i}^{p-1} \mathbf{x}_{N,K} \mathbf{w}_{N;i} \mathbf{y}_N & \mathbf{w}_{N,0;i}^{p-1} \mathbf{x}_{N,K} \mathbf{w}_{N;i} \mathbf{x}_{N,1} & \dots & \mathbf{w}_{N,0;i}^{p-1} \mathbf{x}_{N,K} \mathbf{w}_{N;i} \mathbf{x}_{N,K} & \dots & \mathbf{w}_{N,0;i}^{p-1} \mathbf{x}_{N,K} x_{N,K;i} \\
\vdots & \vdots & & \vdots & & \vdots \\
\mathbf{w}_{N,0;i} \mathbf{x}_{N,K} \mathbf{w}_{N;i} \mathbf{y}_N & \mathbf{w}_{N,0;i} \mathbf{x}_{N,K} \mathbf{w}_{N;i} \mathbf{x}_{N,1} & \dots & \mathbf{w}_{N,0;i} \mathbf{x}_{N,K} \mathbf{w}_{N;i} \mathbf{x}_{N,K} & \dots & \mathbf{w}_{N,0;i} \mathbf{x}_{N,K} x_{N,K;i} \\
\vdots & \vdots & & \vdots & & \vdots \\
x_{N,K;i} \mathbf{w}_{N;i} \mathbf{y}_N & x_{N,K;i} \mathbf{w}_{N;i} \mathbf{x}_{N,1} & \dots & x_{N,K;i} \mathbf{w}_{N;i} \mathbf{x}_{N,K} & \dots & x_{N,K;i}^2
\end{bmatrix}. \tag{A-1}$$

If, in expectation, the columns of the matrix $\mathbf{z}_{N;i} \mathbf{d}_{N;i}^\top$ are linearly independent for all i, it follows that $\mathbb{E}[N^{-1} \sum_{i \in \mathcal{I}_N} \mathbf{z}_{N;i} \mathbf{d}_{N;i}^\top]$ has full column rank. We check the linear independence across columns for a generic row from the matrix in (A-1). First, note that for all the components that involve the outcome variable $\mathbf{y}_N$, the linearity assumption in 2 implies that, for $K = 1$,

$$\mathbf{W}_N \mathbf{y}_N = \gamma_0 \mathbf{W}_N \mathbf{x}_N + \pi_0 \sum_{r=0}^{\infty} \beta_0^r \mathbf{W}_N^{r+2} \mathbf{x}_N + \sum_{r=0}^{\infty} \beta_0^r \mathbf{W}_N^{r+1} \varepsilon_N,$$

where $\pi_0 = (\gamma_0 \beta_0 + \delta_0)$. Thus, all elements in the first column involve infinite powers of the endogenous adjacency matrix $\mathbf{W}_N$. In particular, it follows that for an arbitrary individual i and any number of regressors K,

$$\begin{aligned} \mathbf{w}_{N;i} \mathbf{y}_N =& \gamma_{0,1} \mathbf{w}_{N;i} \mathbf{x}_{N,1} + \cdots + \gamma_{0,K} \mathbf{w}_{N;i} \mathbf{x}_{N,K} + \pi_{0,1} \sum_{r=0}^{\infty} \beta_0^r \mathbf{w}_{N;i} \mathbf{W}_N^{r+1} \mathbf{x}_{N,1} + \dots \\ &+ \pi_{0,K} \sum_{r=0}^{\infty} \beta_0^r \mathbf{w}_{N;i} \mathbf{W}_N^{r+1} \mathbf{x}_{N,K} + e_i, \end{aligned} \tag{A-2}$$

where $e_i = \sum_{r=0}^{\infty} \beta_0^r \mathbf{w}_{N;i} \mathbf{W}_N^r \varepsilon_N$ and $\pi_{0,k} = (\gamma_{0,k} \beta_0 + \delta_{0,k})$ for some $k \in \{1, \dots, K\}$. Choose an arbitrary row k from the first K rows in equation (A-1). Taking expectations with respect to the joint distribution $f_{\mathbf{X}_N, \mathcal{G}_N, \mathcal{G}_{N,0}, \boldsymbol{\varepsilon}_N}(\mathbf{X}, \mathbf{g}, \mathbf{g}_0, \boldsymbol{\varepsilon})$, the expected value for row k is given by the vector

$$[\mathbb{E}[\mathbf{w}_{N,0;i}^p \mathbf{x}_{N,k} \mathbf{w}_{N;i} \mathbf{y}_N], \dots, \mathbb{E}[\mathbf{w}_{N,0;i}^p \mathbf{x}_{N,k} \mathbf{w}_{N;i} \mathbf{x}_{N,K}], \dots, \mathbb{E}[\mathbf{w}_{N,0;i}^p \mathbf{x}_{N,k} x_{N,K;i}]].$$

By definition of conditional expectations and considering the discrete nature of random networks, for any two aggressors k and ℓ, we can write

$$\begin{aligned} \mathbb{E}[\mathbf{w}_{N,0;i}^p \mathbf{x}_{N,k} \mathbf{w}_{N;i} \mathbf{x}_{N,\ell}] = \sum_{\mathbf{w}_0 : \mathbf{g}_0 \in \mathscr{G}_0} \int_{\mathbf{x}_k : \mathbf{X} \in \mathscr{X}} & \mathbf{w}_0^p \mathbf{x}_k \mathbb{E}[\mathbf{w}_{N;i} \mathbf{x}_{N,\ell} \mid \mathbf{w}_{N,0;i} = \mathbf{w}_0, \mathbf{x}_{N,k} = \mathbf{x}_k] \\ & f_{\mathbf{x}_{N,k}}(\mathbf{x}_k) \Pr(\mathcal{G}_{N,0} = \mathbf{g}_0) d\mathbf{x}_k, \end{aligned} \tag{A-3}$$

where $f_{\mathbf{x}_{N,k}}(\mathbf{x}_k) \Pr(\mathcal{G}_{N,0} = \mathbf{g}_0)$ represents the product of the marginal distributions of the kth regressors and the exogenous network. We can represent the distribution of the regressors and the exogenous network by the products of the marginals because of the independence guaranteed by the properties of randomization. From Assumption A.10, $\mathbb{E}[\mathbf{w}_{N,0;i}^p \mathbf{x}_{N,k} \mathbf{w}_{N;i} \mathbf{x}_{N,\ell}]$ exists, which implies that the conditional expectations defined in (A-3) also exist. Choose arbitrary values of $\mathbf{x}_k$ and $\mathbf{g}_0$ such that $\mathbb{E}[\mathbf{w}_{N;i} \mathbf{x}_{N,\ell} \mid \mathbf{w}_{0;i} = \mathbf{w}_0, \mathbf{x}_k = \mathbf{x}_k] \neq 0$, and that occur with positive probability in $f_{\mathbf{x}_{N,k}}(\mathbf{x}_k)$ and $\Pr(\mathcal{G}_{N,0} = \mathbf{g}_0)$.

We can collect all the values related to the same arbitrary regressor $\mathbf{x}_\ell$ from the expectation vector $[\mathbb{E}[\mathbf{w}^p_{N,0;i}\mathbf{x}_{N,k}\mathbf{w}_{N;i}\mathbf{y}_N], \ldots, \mathbb{E}[\mathbf{w}^p_{N,0;i}\mathbf{x}_{N,k}\mathbf{w}_{N;i}\mathbf{x}_{N,K}], \ldots, \mathbb{E}[\mathbf{w}^p_{N,0;i}\mathbf{x}_{N,k}x_{N,K;i}]]$, which, after replacing $\mathbf{w}_{N;i}\mathbf{y}_N$ with the expression in (A-2) and considering the assumption A.15, is given by

$$[\gamma_{0,\ell}\mathbf{w}^p_0\mathbf{x}_k\mathbb{E}[\mathbf{w}_{N;i}\mathbf{x}_{N,\ell} \mid \mathbf{w}_0, \mathbf{x}_k] + \pi_{0,\ell}\mathbf{w}^p_0\mathbf{x}_k\sum_{r=0}^{\infty}\beta^r\mathbb{E}[\mathbf{w}_{N;i}\mathbf{W}_N^{r+1}\mathbf{x}_{N,\ell} \mid \mathbf{w}_0, \mathbf{x}_k], \tag{A-4}$$
$$\mathbf{w}^p_0\mathbf{x}_k\mathbb{E}[\mathbf{w}_{N;i}\mathbf{x}_{N,\ell} \mid \mathbf{w}_0, \mathbf{x}_k], \mathbf{w}^p_0\mathbf{x}_k\mathbb{E}[x_{N,\ell;i} \mid \mathbf{w}_0, \mathbf{x}_k]].$$

The three components of the vector in (A-4) are linearly dependent if and only if there exist three constants a, b and c different from zero such that

$$\mathbb{E}[(a\gamma_{0,\ell} + b)\mathbf{w}_{N;i}\mathbf{x}_{N,\ell} + cx_{N,\ell;i} + a\pi_{0,\ell}\beta\mathbf{w}_{N;i}\mathbf{W}_N\mathbf{x}_{N,\ell} + \tag{A-5}$$
$$a\pi_{0,\ell}\beta^2\mathbf{w}_{N;i}\mathbf{W}_N^2\mathbf{x}_{N,\ell} + \cdots \mid \mathbf{w}_0, \mathbf{x}_k] = 0,$$

where the dots represent the infinite sum on r. Under the assumption that $\pi_{0,\ell} \neq 0$, the only way in which equation (A-5) holds for constant a, b, and c different from zero is if the matrices $\mathbf{I}_N, \mathbf{W}_N, \mathbf{W}_N^2, \ldots$ are linearly dependent and there exists $\mathbf{W}_N^r$ such that $x_{i,\ell} = \mathbf{w}^r_{N;i}\mathbf{x}_\ell$. If $\pi_{0,\ell} = 0$, clearly the first and second components of the vector are linearly dependent. Therefore, Assumptions A.11 and A.12 imply that the components of the vector in (A-4) are linearly independent. We arbitrarily chose the regressors k and ℓ. Then, under Assumption A.9 all the regressors are linearly independent, which implies that all the components of the vector

$$[\mathbb{E}[\mathbf{w}^p_{N,0;i}\mathbf{x}_{N,k}\mathbf{w}_{N;i}\mathbf{y}_N], \ldots, \mathbb{E}[\mathbf{w}^p_{N,0;i}\mathbf{x}_{N,k}\mathbf{w}_{N;i}\mathbf{x}_{N,K}], \ldots, \mathbb{E}[\mathbf{w}^p_{N,0;i}\mathbf{x}_{N,k}x_{N,K;i}]]$$

are linearly independent. The same result follows for the vectors of the type

$$[\mathbb{E}[x_{N,k;i}\mathbf{w}_{N;i}\mathbf{y}_N], \ldots, \mathbb{E}[x_{N,k;i}\mathbf{w}_{N;i}\mathbf{x}_{N,k}], \ldots, \mathbb{E}[x^2_{N,k;i}]]$$

by conditioning on the arbitrary regressor $x_{N,k;i}$ for nonzero rows. To show that the rows are linearly independent, we can use an analogous approach considering arbitrary values of $\mathbf{w}_i$ and $\mathbf{x}_K$. It is straightforward to see that under the column rank assumption on all matrices of regressors, Assumption A.13 implies the result. Finally, given that we showed linear independence conditions for nonzero rows, we need to show that there are at least

$2K+1$ rows different from zero. The rank condition on the matrix of regressors implies that we only need to focus on combinations of connections in $\mathbf{W}_{N,0}$ and $\mathbf{W}_N$ that can make $\mathbb{E}[\mathbf{w}^p_{0,N,i}\mathbf{x}_{N,\ell}\mathbf{w}_{N,i}\mathbf{x}_{N,k}] = 0$ for any value of $\mathbf{x}_\ell$ and $\mathbf{x}_k$ and some value of p. First, note that

$$\mathbb{E}[\mathbf{w}^p_{N,0;i}\mathbf{x}_{N,\ell}\mathbf{w}_{N;i}\mathbf{x}_{N,k}] = \int_{\mathbf{x}_k:\mathbf{X}\in\mathscr{X}}\int_{\mathbf{x}_\ell:\mathbf{X}\in\mathscr{X}} \mathbb{E}[\mathbf{w}^p_{N,0;i}\mathbf{x}_\ell\mathbf{w}_i\mathbf{x}_N] f_{\mathbf{x}_{N,k}\mathbf{x}_{N,\ell}}(\mathbf{x}_k,\mathbf{x}_\ell)d\mathbf{x}_k d\mathbf{x}_\ell, \quad \text{(A-6)}$$

where $f_{\mathbf{x}_{N,k}\mathbf{x}_{N,\ell}}$ is the joint probability of $\mathbf{x}_{N,k}$ and $\mathbf{x}_{N,\ell}$. Take some arbitrary values $\mathbf{x}_k \neq 0$ and $\mathbf{x}_\ell \neq 0$ with positive probability in $f_{\mathbf{x}_{N,k}\mathbf{x}_{N,\ell}}$. It follows that

$$\mathbb{E}[\mathbf{w}^p_{N,0;i}\mathbf{x}_\ell\mathbf{w}_i\mathbf{x}_{N,k}] = \sum_{\mathbf{w}_{0;i}:\mathbf{g}_0\in\mathscr{G}_0}\sum_{\mathbf{w}_i:\mathbf{g}\in\mathscr{G}} \mathbf{w}^p_{0;i}\mathbf{x}_\ell\mathbf{w}_i\mathbf{x}_k \Pr(\mathcal{G}_N = \mathbf{g}, \mathcal{G}_{N,0} = \mathbf{g}_0). \quad \text{(A-7)}$$

The only way in which the equation (A-7) can equal zero for different values of p, even when $\mathbf{x}_k \neq 0$ and $\mathbf{x}_\ell \neq 0$, is if the linear combination of $\mathbf{w}^p_{0;i}\mathbf{x}_\ell\mathbf{w}_i\mathbf{x}_k$ for different values of $\mathbf{w}^p_{0;i}$ and $\mathbf{w}_i$ weighted by their respective probabilities equals zero. Therefore, Assumption A.14 guarantees the existence of at least $3K$ rows different from zero. ■

B Proofs of Main Results

Proof of Theorem 1. First, note that Assumption 2 guarantees that the solution for model (3.1) exists. Assumption 3 guarantees that the system of equations $\mathbb{E}[\boldsymbol{m}_N(\boldsymbol{\theta})] = \mathbf{0}_K$ are not trivially satisfied by making all individuals $i \in \mathcal{I}_N$ isolated. We show that the moment condition equation has a unique root at $\boldsymbol{\theta}_0 = (\alpha_0, \beta_0, \boldsymbol{\delta}_0^\top, \boldsymbol{\gamma}_0^\top)^\top$. In particular, we show that there cannot be any other $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ different from $\boldsymbol{\theta}_0$ for which the moment condition is satisfied. Choose an arbitrary vector of parameters $\boldsymbol{\theta} \in \boldsymbol{\Theta}$, such that $\mathbb{E}[\mathbf{m}(\boldsymbol{\theta})] = 0$. Assumptions 1 and 2 imply that $\mathbb{E}[N^{-1}\sum_{i\in\mathcal{I}_N}\mathbf{z}_{N;i}(y_{N;i} - \mathbf{d}^\top_{N;i}\boldsymbol{\theta})] = \mathbf{0}_K$. It follows that $\mathbb{E}[\sum_{i\in\mathcal{I}_N}\mathbf{z}_{N;i}\mathbf{d}^\top_{N;i}](\boldsymbol{\theta}_0 - \boldsymbol{\theta}) + \mathbb{E}[\sum_{i\in\mathcal{I}_N}\mathbf{z}_{N;i}\varepsilon_{N;i}] = \mathbf{0}_K$ and $\mathbb{E}[\sum_{i\in\mathcal{I}_N}\mathbf{z}_{N;i}\mathbf{d}^\top_{N;i}]\,(\boldsymbol{\theta}_0 - \boldsymbol{\theta}) = \mathbf{0}_K$, given that N is arbitrarily large, but finite. Under Assumption 3, it follows that $\mathbb{E}[\boldsymbol{m}(\boldsymbol{\theta})] = \mathbf{0}_K$ if and only if $\boldsymbol{\theta}_0 = \boldsymbol{\theta}$. ■

Proof of Theorem 2. The GMM estimator in (4.1) in the main text can be written as

$$\widehat{\boldsymbol{\theta}}_{\text{GMM}} = \boldsymbol{\theta} + (n^{-1}\mathbf{D}_n^\top\mathbf{Z}_n\mathbf{A}_n n^{-1}\mathbf{Z}_n^\top\mathbf{D}_n)^{-1}n^{-1}\mathbf{D}_n^\top\mathbf{Z}_n\mathbf{A}_n n^{-1}\mathbf{Z}_n^\top\boldsymbol{\varepsilon}_n. \quad \text{(B-1)}$$

By construction, it is assumed that the matrix $\mathbf{A}_n$ converges to the full-rank matrix $\mathbf{A}_N$ as $n \to \infty$. From Corollary C.1, $n^{-1}\mathbf{Z}_n^\top\mathbf{D}_n$ converges to the population quantity

$\mathbb{E}[N^{-1}\sum_{i\in\mathcal{I}_N}\mathbf{z}_{N;i}\mathbf{d}_{N;i}^\top]$, which is finite given Assumption 3. Finally, Corollary C.2 shows that $n^{-1}\mathbf{Z}_n^\top\boldsymbol{\varepsilon}_n(\boldsymbol{\theta})$ converges to $\mathbb{E}[N^{-1}\sum_{i\in\mathcal{I}_N}\mathbf{z}_{N;i}\varepsilon_{N;i}(\boldsymbol{\theta})]=0$. It then follows that $\widehat{\boldsymbol{\theta}}_{\text{GMM}}=\boldsymbol{\theta}+o_p(1)$ as $n\to\infty$. For asymptotic normality, note that from (B-1)

$$\sqrt{n}(\widehat{\boldsymbol{\theta}}_{\text{GMM}}-\boldsymbol{\theta})=(n^{-1}\mathbf{D}_n^\top\mathbf{Z}_n\mathbf{A}_n n^{-1}\mathbf{Z}_n^\top\mathbf{D}_n)^{-1}n^{-1}\mathbf{D}_n^\top\mathbf{Z}_n\mathbf{A}_n\times n^{-1/2}\mathbf{Z}_n^\top\boldsymbol{\varepsilon}_n.$$

Let $\mathbf{Q}_{zx}=\mathbb{E}[N^{-1}\sum_{i\in\mathcal{I}_N}\mathbf{z}_{N;i}\mathbf{d}_{N;i}^\top]$. Then from Corollary C.1 and Lemma C.3, it follows that

$$\sqrt{n}\left(\widehat{\boldsymbol{\theta}}_{\text{GMM}}-\boldsymbol{\theta}\right)\xrightarrow{d}\left[\mathbf{Q}_{zx}^\top\mathbf{A}_N\mathbf{Q}_{zx}\right]^{-1}\mathbf{Q}_{zx}^\top\mathbf{A}_N\times\mathcal{N}(\mathbf{0},\boldsymbol{\Omega}_N).$$

The result then follows. The efficient variance-covariance matrix in (4.4) is derived from standard matrix algebra calculations. ■

C Auxiliary Results

All the results in this section, and consequently in section B, are derived conditional on the sequence of networks $\{\mathcal{G}_n\}$. For simplicity in notation, we have omitted explicit conditioning on expectations, but it is important to note that all expectations are taken with respect to the conditional distribution $f_{\mathbf{X}_N,\boldsymbol{\varepsilon}_N|\mathcal{G}_N}$.

Lemma C.1 *Let Assumption 4 hold for* $\{\mathbf{r}_{n;i}\}_{n\geq 1}$, $i\in\mathcal{I}_n$ *and define* $R_{n;i,j}=f_{q,\ell}(\mathbf{r}_{n,\{i,j\}})\equiv r_{n;i,q}r_{n;j,\ell}$ *and* $R_{n;h,s}=g_{q',\ell'}(\mathbf{r}_{n,\{h,s\}})\equiv r_{n;h,q'}r_{n;s,\ell'}$ *for* $i,j,h,s\in\mathcal{I}_n$, *where* q, q', ℓ, *and* ℓ' *are components of the vector* $\mathbf{r}_{n;i}$. *Let Assumption 5 hold for* $R_{i,j}$ *and* $R_{h,s}$; *then*

$$|\text{cov}\,(R_{n;i,j},R_{n;h,s})|\leq 2\bar{\lambda}_{n,d}(C+16)\times 4\,(\pi_1+\tilde{\gamma}_1)\,(\pi_2+\tilde{\gamma}_2)\,\underline{\lambda}_{n,d}^{1-p_f-p_g}, \qquad \text{(C-1)}$$

where $\underline{\lambda}_{n,d}=\lambda_{n,d}\wedge 1$, $\bar{\lambda}_{n,d}=\lambda_{n,d}\vee 1$, $\pi_1=\|\mathbf{r}_{n;i}\|_{p_{f,i}}\|\mathbf{r}_{n;j}\|_{p_{f,j}}$, $\pi_2=\|\mathbf{r}_{n;h}\|_{p_{f,h}}\|\mathbf{r}_{n;s}\|_{p_{f,s}}$, $\widetilde{\gamma}_1=\max\{\|\mathbf{r}_{n;i}\|_{p_{f,i}+p_{f,j}},\|\mathbf{r}_{n;j}\|_{p_{f,i}+p_{f,j}}\}$; $\widetilde{\gamma}_2=\max\{\|\mathbf{r}_{n;h}\|_{p_f},\|\mathbf{r}_{n;s}\|_{p_g}\}$, *where* $p_f=1/p_{f,i}+1/p_{f,j}$ *and* $p_g=1/p_{g,h}+1/p_{g,s}$, *where the constant* C *is the same as in Assumption 4. The indexes* i,j,h,s *and the components* q, q', ℓ, ℓ' *may or may not be the same.*

Proof. Define the increasing continuous functions $h_1(x)$ and $h_2(x)$ as in Theorem A.2 in Kojevnikov, Marmer, and Song (2021, Appendix A, pp. 899-907) as $h_1(x)=h_2(x)=x$. Note that the functions $f_{q,\ell}$ and $g_{q',\ell'}$ are continuous, and their truncated version of the form $\varphi_{K_1}\circ f\circ\varphi_{h_1}(K_2)$ and $\varphi_{K_1}\circ g\circ\varphi_{h_1}(K_2)$ for all $K\in(0,\infty)^2$ is in $\mathscr{L}_{Q+1,2}$. Assumption 5 guarantees the existence of the moments defining $\widetilde{\gamma}_1$ and $\widetilde{\gamma}_2$. Then, Theorem A.2 in Kojevnikov, Marmer, and Song (2021, Appendix A, pp. 899-907) applies to this setting (see also Corollary A.2. in Appendix A in Kojevnikov, Marmer, and Song, 2021, pp. 899-907). ∎

Lemma C.2 (LLN for Products of ψ-dependent Random Variables) *Let Assumptions 4 – 7 hold, define $R_{n;i,j} \equiv r_{n;i,q} r_{n;j,\ell}$, and let $w^*_{i,j}$ be weights between zero and one. Form $\{R_{n;i,j}\}_{i\in\mathcal{I}_n, j\in\mathcal{I}_i}$, where $\mathcal{I}_i \subset \mathcal{I}_n$ is a set of indexes defined for each $i \in \mathcal{I}_n$, which can be empty, equal to the union of individual i's connections in the networks $\mathcal{G}_n$ and $\mathcal{G}_{n,0}$, or equal to $\mathcal{P}_n(i,1)$. Then, as $n \to \infty$,*

$$\left\| \frac{1}{n} \sum_{i\in\mathcal{I}_n} \sum_{j\in\mathcal{I}_i} w^*_{i,j} \left(R_{n;i,j} - \mathbb{E}\left[R_{n;i,j}\right]\right) \right\|_1 \longrightarrow 0.$$

Proof. Using the same approach as Jenish and Prucha (2009) and Kojevnikov, Marmer, and Song (2021), let the censoring function $\varphi_k(x) = (-K) \vee (K \wedge x)$ be such that, for some $k > 0$,

$$R_{n;i,j} = R^{(k)}_{n;i,j} + \tilde{R}^{(k)}_{n;i,j},$$

where $R^{(k)}_{n;i,j} = \varphi_k(R_{n;i,j})$ and $\tilde{R}^{(k)}_{n;i,j} = R_{n;i,j} - \varphi_k(R_{n;i,j}) = (R_{n;i,j} - \text{sgn}(R_{n;i,j})k)\mathbb{1}\{|R_{n;i,j}| > k\}$. Let $\|X\|_k = (\mathbb{E}[|X|^k)^{1/k}$ for $k \in [1,\infty)$. Therefore, following the previous definition, we apply the triangle inequality to get

$$\begin{aligned}\left\| \frac{1}{n} \sum_{i\in\mathcal{I}_n} \sum_{j\in\mathcal{I}_i} w^*_{i,j} \left(R_{n;i,j} - \mathbb{E}\left[R_{n;i,j}\right]\right) \right\|_1 \leq & \left\| \frac{1}{n} \sum_{i\in\mathcal{I}_n} \sum_{j\in\mathcal{I}_i} w^*_{i,j} \left(R^{(k)}_{n;i,j} - \mathbb{E}\left[R^{(k)}_{n;i,j}\right]\right) \right\|_1 \\ & + \left\| \frac{1}{n} \sum_{i\in\mathcal{I}_n} \sum_{j\in\mathcal{I}_i} w^*_{i,j} \left(\tilde{R}^{(k)}_{n;i,j} - \mathbb{E}\left[\tilde{R}^{(k)}_{n;i,j}\right]\right) \right\|_1 .\end{aligned}$$

From Assumption 7, note that the expectation on the second term of the previous equation is bounded by $\mathbb{E}[|\tilde{R}^{(k)}_{n;i,j}|] = \mathbb{E}[|\tilde{R}^{(k)}_{n;i,j}|\mathbb{1}\{|R_{n;i,j}| > k\}] \leq 2\mathbb{E}[|R_{n;i,j}|\mathbb{1}\{|R_{n;i,j}| > k\}]$. Following the arguments as in Kojevnikov, Marmer, and Song (2021), the second component of the right-hand side of the above equation is bounded by $\sup_{n\geq 1} \max_{i\in\mathcal{I}_n} \mathbb{E}\left[|R_{n;i,j}|\, \mathbb{1}\left\{|R_{n;i}| > k\right\}\right]$, where $\lim_{k\to\infty} \sup_{n\geq 1} \max_{i\in\mathcal{I}_n} \mathbb{E}\left[|R_{n;i,j}|\, \mathbb{1}\left\{|R_{n;i,j}| > k\right\}\right] = 0$. Focusing on the first component of the right-hand side, by Lyapunov's inequality, it follows that

$$\begin{aligned}\left\| \frac{1}{n} \sum_{i\in\mathcal{I}_n} \sum_{j\in\mathcal{I}_i} w^*_{i,j} \left(R^{(k)}_{n;i,j} - \mathbb{E}\left[R^{(k)}_{n;i,j}\right]\right) \right\|_1 \leq & \left\| \frac{1}{n} \sum_{i\in\mathcal{I}_n} \sum_{j\in\mathcal{I}_i} w^*_{i,j} \left(R^{(k)}_{n;i,j} - \mathbb{E}\left[R^{(k)}_{n;i,j}\right]\right) \right\|_2 \\ = & \frac{1}{n} \sqrt{\text{var}\left(\sum_{i\in\mathcal{I}_n} \sum_{j\in\mathcal{I}_i} w^*_{i,j} R^{(k)}_{n;i,j} \right)},\end{aligned} \tag{C-2}$$

where (C-2) is an expression for the standard deviation of $\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{I}_i} w^*_{i,j}R^{(k)}_{n;i,j}$. Note that

$$\text{var}\left(\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{I}_i} w^*_{i,j}R^{(k)}_{n;i,j}\right) = \sum_{i\in\mathcal{I}_n}\text{var}\left(\sum_{j\in\mathcal{I}_i} w^*_{i,j}R^{(k)}_{n;i,j}\right) + \sum_{i\neq h\in\mathcal{I}_n}\text{cov}\left(\sum_{j\in\mathcal{I}_i} w^*_{i,j}R^{(k)}_{n;i,j}, \sum_{s\in\mathcal{I}_h} w^*_{h,s}R^{(k)}_{n;h,s}\right).$$

The variance part of the previous equation can be further expressed as

$$\begin{aligned}
\text{var}\left(\sum_{j\in\mathcal{I}_i} w^*_{i,j}R^{(k)}_{n;i,j}\right) &= \sum_{j\in\mathcal{I}_i} w^{*2}_{i,j}\text{var}(R^{(k)}_{n;i,j}) + \sum_{j\neq s\in\mathcal{I}_i} w^*_{i,j}w^*_{i,s}\text{cov}(R^{(k)}_{n;i,j}, R^{(k)}_{n;i,s}), \\
&\leq C\sum_{j\in\mathcal{I}_i} w^{*2}_{i,j} + \sum_{j\in\mathcal{I}_i}\sum_{d\geq 1}\sum_{s\in\mathcal{P}_n(j,d)\cap\mathcal{I}_i} |\text{cov}(R^{(k)}_{n;i,j}R^{(k)}_{n;i,s})|, \\
&\leq C\sum_{j\in\mathcal{I}_i} w^{*2}_{i,j} + \psi_{1,1}\left(\varphi_k,\varphi_k\right)\sum_{d\geq 1}\lambda_{n,d}\sum_{j\in\mathcal{I}_i}\left|\mathcal{P}_n(j,d)\right|,
\end{aligned} \tag{C-3}$$

where the second inequality follows from $w^*_{i,j}, w^*_{i,s}\in[0,1]$. In the first term of the second inequality, C represents any generic constant due to the fact that after the initial partition of $R_{n;i,j}$, the variance of $R^{(k)}_{n;i,j}$ is bounded. The last inequality follows from two reasons. First, from Lemma C.1 under Assumptions 4 and 5, $|\text{cov}(R^{(k)}_{n;i,j}R^{(k)}_{n;i,s})| \leq \psi_{1,1}\left(\varphi_k,\varphi_k\right)\lambda_{n,d}$ for $d_n(i,j)=d$ and φ_k is a bounded function with $\text{Lip}\left(\psi_k\right)=1$. Second, the set of indexes $\mathcal{P}_n(j,d)$ is such that $\mathcal{P}_n(j,d)\cap\mathcal{I}_i\subset\mathcal{P}_n(j,d)$. The covariance component can be written as

$$\begin{aligned}
\text{cov}\left(\sum_{j\in\mathcal{I}_i} w^*_{i,j}R^{(k)}_{n;i,j}, \sum_{s\in\mathcal{I}_h} w^*_{h,s}R^{(k)}_{n;h,s}\right) &= \sum_{j\in\mathcal{I}_i}\sum_{s\in\mathcal{I}_h} w^*_{i,j}w^*_{h,s}\text{cov}(R^{(k)}_{n;i,j}, R^{(k)}_{n;h,s}), \\
&\leq \sum_{j\in\mathcal{I}_i}\sum_{d\geq 1}\sum_{s\in\mathcal{P}_n(j,d)\cap\mathcal{I}_h} |\text{cov}(R^{(k)}_{n;i,j}R^{(k)}_{n;h,s})|, \\
&\leq \psi_{1,1}\left(\varphi_k,\varphi_k\right)\sum_{d\geq 1}\lambda_{n,d}\sum_{j\in\mathcal{I}_i}\left|\mathcal{P}_n(j,d)\right|,
\end{aligned} \tag{C-4}$$

where the second and third inequalities follow from the same principles already discussed in the previous paragraph. It follows from Equations (C-3) and (C-4) that the total variance of $\sum_{j\in\mathcal{I}_i} w^*_{i,j}R^{(k)}_{n;i,j}$ can be bounded by

$$\text{var}\left(\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{I}_i} w^*_{i,j} R^{(k)}_{n;i,j}\right) = C\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{I}_i} w^{*2}_{i,j} + 2\psi_{1,1}\left(\varphi_k,\varphi_k\right)\sum_{i\in\mathcal{I}_n}\sum_{d\geq 1}\lambda_{n,d}\sum_{j\in\mathcal{I}_i}\left|\mathcal{P}_n(j,d)\right|, \tag{C-5}$$
$$= C\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{I}_i} w^{*2}_{i,j} + 2\psi_{1,1}\left(\varphi_k,\varphi_k\right)\sum_{d\geq 1}\lambda_{n,d}\sum_{i\in\mathcal{I}_n}\left|\mathcal{P}_n(j,d)\right|,$$
$$\leq n\left(C\bar{\mathcal{I}}_n + 2\psi_{1,1}\left(\varphi_k,\varphi_k\right)\sum_{d\geq 1}\bar{D}_n(d)\lambda_{n,d}\right),$$

where $\bar{\mathcal{I}}_n = n^{-1}\sum_{i\in\mathcal{I}_n}|\mathcal{I}_i|$ and the inequality follows because $w^{*2}_{i,j}\in[0,1]$. The set $\mathcal{I}_i$ can either be empty, equal to the union of individual i's connections in the networks $\mathcal{G}_n$ and $\mathcal{G}_{n,0}$, or equal to $\mathcal{P}_n(i,1)$ (individual i's connections in network $\mathcal{G}_n$). Note that, for any of the three cases, $|\mathcal{I}_i|\leq|\mathcal{P}_n(i,1)|$ for all i. Also, $\sum_{i\in\mathcal{I}_n}|\mathcal{P}_n(i,1)|\lambda_{n,1}\leq\sum_{d\geq 1}\bar{D}_n(d)\lambda_{n,d}$, which converges in probability to zero by Assumption 6. It follows that $n^{-1}\bar{\mathcal{I}}_n\xrightarrow{\text{p}}0$. Therefore,

$$\left\|\frac{1}{n}\sum_{i\in\mathcal{I}_{n,m}}\sum_{j\in\mathcal{I}_i} w^*_{i,j}\left(R^{(k)}_{n;i,j} - \mathbb{E}\left[R^{(k)}_{n;i,j}\right]\right)\right\|_1 \leq \left(n^{-1}C\bar{\mathcal{I}}_n + 2\psi_{1,1}n^{-1}\sum_{d\geq 1}\bar{D}_n(d)\lambda_{n,d}\right)^{1/2}. \tag{C-6}$$

The result follows from $n^{-1}\bar{\mathcal{I}}_n\xrightarrow{\text{p}}0$ and $n^{-1}\sum_{d\geq 1}\bar{D}_n(d)\lambda_{n,d}\xrightarrow{\text{p}}0$ under Assumption 6. ■

Corollary C.1 (LLN for Instruments and Regressors) *Let Assumptions 4 to 7 hold. Then,*

$$\left\|\frac{1}{n}\sum_{i\in\mathcal{I}_n}\left(\mathbf{z}_{n;i}\mathbf{d}^\top_{n;i} - \mathbb{E}[\mathbf{z}_{n;i}\mathbf{d}^\top_{n;i}]\right)\right\|_1 \longrightarrow 0.$$

Proof. There are four different types of components in the matrix $\mathbf{Z}_n^\top\mathbf{D}_n$ formed by summation of products of: (1) Non-network regressors of the form $x_{n;i,q}x_{n;i,\ell}$; (2) Network regressors of the form $\mathbf{w}_{n,0;i}\mathbf{x}_{n,q}\mathbf{w}_{n;i}\mathbf{x}_{n;\ell}$; (3) Network and non-network regressors of the form $\mathbf{w}_{n,0;i}\mathbf{x}_{n,q}\mathbf{x}_{n;i,\ell}$; and (4) Network regressors and network outcomes of the form $\mathbf{w}_{n,0;i}\mathbf{x}_{n,q}\mathbf{w}_{n;i}\mathbf{y}_n$ [and the versions of (2) and (3) with $\mathbf{w}^p_{n,0;i}$ instead of $\mathbf{w}_{n,0;i}$]. The LLN follows from Lemma C.2 by choosing $\mathcal{I}_i=\{\emptyset\}$ for (1), $\mathcal{I}_i$ as the union of individual i's connections in the networks $\mathcal{G}_n$ and $\mathcal{G}_{n,0}$ in (2), and $\mathcal{I}_i=\mathcal{P}_n(i,1)$ for (3). For (4), note that

$$\mathbb{E}[\mathbf{W}_N\mathbf{y}] = \gamma_0\mathbf{W}_N\mathbb{E}[\mathbf{x}_N] + (\gamma_0\beta_0+\delta_0)\sum_{p=0}^{\infty}\beta_0^k\mathbf{W}_N^{p+2}\mathbb{E}[\mathbf{x}_N], \tag{C-7}$$

where, again, the expectation is taken conditional on $\mathcal{G}_N$. By choosing $\mathcal{I}_i$ to be the union of individual i's connections in the network $\mathcal{G}$ and the set of individuals at distance p from i (for all $p \in \mathbb{R}_+$), Lemma C.2 applies for all values in the infinite sum formed by $\mathbf{w}_{n,0;i}\mathbf{x}_{n,q}\mathbf{w}_{n;i}\mathbf{y}_n$ after replacing $\mathbf{w}_{n;i}\mathbf{y}_n$ from Equation (C-7) [the same argument holds for (2) and (3) when using $\mathbf{w}^p_{n,0;i}$ instead of $\mathbf{w}_{n,0;i}$]. Given that each component of the sum converges to a finite expectation, the infinite sum of finite expectations is also finite given the restriction on the parameters β_0 from Assumption 2, thus completing the proof. ■

Corollary C.2 (LLN for Instruments and Errors) *Let Assumptions 4 to 7 hold, then*

$$\left\| \frac{1}{n} \sum_{i \in \mathcal{I}_n} \left(\mathbf{z}_{n;i}\varepsilon^\top_{n;i} - \mathbb{E}[\mathbf{z}_{n;i}\varepsilon^\top_{n;i}] \right) \right\|_1 \longrightarrow 0.$$

Proof. Given that $\mathbf{r}_{n;i} = [\mathbf{x}_{n;i}, \varepsilon_{n;i}]$ and $\mathbf{z}_{n;i}$ can be divided into both network and nonnetwork components, the proof of this result is analogous to that of Corollary C.1 (1) and (3).
■

Corollary C.3 (Finite Variance) *Define* $\mathbf{S}_n = \mathbf{Z}^\top_n \boldsymbol{\varepsilon}_n$ *and* $\boldsymbol{\Omega}_n = \mathrm{var}(n^{-1/2}\mathbf{S}_n)$ *and let Assumptions 4 to 7 hold, then as* $n \to \infty$, $\boldsymbol{\Omega}_n \longrightarrow \boldsymbol{\Omega}_N < \infty$.

Proof. As before, $n^{-1/2}\mathbf{S}_n \equiv n^{-1/2}\sum_{i=1}^n \mathbf{z}_{n;i}\varepsilon_{n;i}$. The bounded covariance assumptions of Lemma C.1 combined with the arguments of Lemma C.2 guarantee that the following limit $\lim_{n\to\infty} n^{-1}\mathrm{var}\left(\sum_{i=1}^n \mathbf{z}_{n;i}\varepsilon_{n;i}\right)$ is finite. In particular, from Equation (C-6), using the appropriate values for $R_{n;i,j}$ and $\mathcal{I}_i$ (see Corollary C.1), it follows that $\mathrm{var}(\sum_{i=1}^n \mathbf{z}_{n;i}\varepsilon_{n;i}) = O_p(1)$. Given that $\boldsymbol{\Omega}_n$ converges to a finite quantity, it follows that $\boldsymbol{\Omega}_n \longrightarrow \boldsymbol{\Omega}_N$, where

$$\boldsymbol{\Omega}_N = \lim_{n\to\infty} n^{-1} \left[\sum_{i=1}^n \mathrm{var}(\mathbf{z}_{n;i}\varepsilon_{n;i}) + \sum_{i \neq j} \mathrm{cov}(\mathbf{z}_{n;i}\varepsilon_{n;i}, \mathbf{z}_{n;j}\varepsilon_{n;j}) \right] < \infty.$$

■

Lemma C.3 (Central Limit Theorem) *Let Assumptions 1 and 4-8 hold and define* $S_n \equiv \sum_{i \in \mathcal{I}_n} z_{n;i,q}\varepsilon_{n;i}$, *where* $z_{n;i,q}$ *is the* qth *entrance of the vector* $\mathbf{z}_{n;i}$. *Then, by the definition of* $\mathbf{z}_{n;i}$ *and Assumption 1,* $\mathbb{E}[z_{n;i,q}\varepsilon_{n;i}] = 0$. *As* $n \to \infty$,

$$\sup_{t \in \mathbf{R}} \left| \mathbf{P}\left\{ \frac{S_n}{\sigma_n} \leq t \right\} - \Phi(t) \right| \longrightarrow 0,$$

where $\sigma_n \equiv var(S_n)$ *and* $\Phi(\cdot)$ *denotes the cumulative distribution function of a standard normal random variable.*

Proof. Let $Y_{n;i} = z_{n;i,q}\varepsilon_{n;i}$. From Lemma C.1, the covariance of any two $Y_{n;i}$ and $Y_{n;j}$ is bounded. The proof then follows from applying the unconditional version of Lemmas A.2 and A.3 in Kojevnikov, Marmer, and Song (2021, Appendix A, pp. 899-907) to $Y_{n;i}$ and S_n/σ_n, respectively. ∎

Lemma C.4 (Multivariate Central Limit Theorem) *Let Assumptions 1 and 4-8 hold. Then, as* $n \to \infty$, $n^{-1/2}\sum_{i=1}^{n} \mathbf{z}_{n;i}\varepsilon_{n;i} \xrightarrow{d} \mathcal{N}(0, \Omega_N)$.

Proof. From Lemma C.3, it follows that $n^{-1/2}\sum_{i=1}^{n} z_{n;i,q}\varepsilon_{n;i} \xrightarrow{d} \mathcal{N}(0, \sigma_n^2)$, while from Lemma C.3, it follows that $\boldsymbol{\Omega}_N$ exists. Therefore, the result follows from an application of the Cramér-Wold device. ∎

D Empirical Application

D.1 Data Description

Our data set was collected between March and May 2011 as part of the Hong Kong Secondary Education Survey in Hong Kong (SESHK). The survey was conducted in the second semester before the final exams and involved three secondary schools with 868 students participating. The sample includes 7th-grade students from all three schools and 8th- and 9th-grade students from one school (g $\in \{7, 8, 9\}$). Each grade within a school is made up of five different sections (cl $\in \{1, \dots, 5\}$).

Table 2 shows the summary statistics for the variables we use in our empirical application. `Math test` corresponds to the first math exam score for each student i. The data set also includes information on a cognitive ability test on five personality measures: `Agreeableness`, `Conscientiousness`, `Extraversion`, `Neuroticism` and `Openness`.[1] In our empirical application, we also include the following variables: `Male` that equals 1 if the student is male, and 0 otherwise. The `Height` for each student is measured in centimeters (cm) and the `Weight` in kilograms (kg). Both the `number of elder and younger siblings` are count variables, and `commute by car / taxi` equals 1 when a student goes to school by car or by taxi.

We also include indicator variables capturing students' engagement at school. For example, the indicator variables `Siblings' Help` and `Parents' Help` take the value of 1 if students receive help from their siblings or parents. We label those variables as 0 otherwise. To capture extracurricular school-related activities, we include the indicator variable `Music`, which equals 1 if students play music, and 0 otherwise.

[1] The numbers in parenthesis in Scale column of Table 2 represent the scales for the tests.

Seatmate and Student Partner Networks

In the survey, students were asked to write lists of up to ten peers from among their schoolmates within the same grade with whom they discussed their problems with schoolwork and who sat next to them in class during the first semester. We use this information to build the study partner and seatmate networks using the following reciprocal peer rule: If students i and j named each other as study partners in the survey, we record an edge in the study partner network. We follow the same process for the seatmate network. Table 3 reports the summary statistics of the network among all students by school.

Seat assignments in the classrooms change several times over a semester, and the changes are decided by the class teacher. Unlike study partners, the seatmate network is based on proximity and imposed by the school. Therefore, the seatmate network is an excellent candidate to be used as the instrumental network $\mathbf{W}_{n,0}$, in our analysis. On the other hand, students can freely choose with whom they study and this decision could be based on unobservable characteristics that also affect exam performance, making the study partner network, $\mathbf{W}_n$, likely endogenous.

As mentioned in Section 6 of the main manuscript, we follow the Linear-in-Means model for social effects. To improve readability, we rewrite the equation 6.1 below,

$$\begin{aligned}
\texttt{math}_{i,\mathrm{s}\times\mathrm{g}\times\mathrm{cl}} \quad = \quad & \alpha + \beta \sum_{j\neq i}^{n} w_{n;i,j}\texttt{math}_{j,\mathrm{s}\times\mathrm{g}\times\mathrm{cl}} \\
& + \sum_{j\neq i}^{n} w_{n;i,j}\texttt{characteristics}'_{j,\mathrm{s}\times\mathrm{g}\times\mathrm{cl}}\delta_{\texttt{characteristics}} \\
& + \sum_{j\neq i}^{n} w_{n;i,j}\texttt{personality}'_{j,\mathrm{s}\times\mathrm{g}\times\mathrm{cl}}\delta_{\texttt{personality}} \\
& + \texttt{characteristics}'_{i,\mathrm{s}\times\mathrm{g}\times\mathrm{cl}}\gamma_{\texttt{characteristics}} + \texttt{personality}'_{i,\mathrm{s}\times\mathrm{g}\times\mathrm{cl}}\gamma_{\texttt{personality}} \\
& + \sum_{\mathrm{s}=1}^{3}\sum_{\mathrm{g}=7}^{9}\sum_{\mathrm{cl}=1}^{5} f_{\mathrm{s}\times\mathrm{g}\times\mathrm{cl}} \times \mathbb{I}\{i \in \mathrm{s}\times\mathrm{g}\times\mathrm{cl}\} + \epsilon_i,
\end{aligned}$$

Estimations results are shown in Table 1, in Section 6 in the main text, and Table 4 (in this section). As mentioned in the manuscript, estimators that do not control for network endogeneity tend to underestimate peer effects. In the context of our simulation results, one possible explanation for the negative bias is the existence of unobserved homophily. If the unobserved variables driving the choice of study partners are negatively correlated with the outcome, we expect the endogeneity bias to underestimate the actual peer effects value.

Students can choose to study with others they find fun for reasons other than learning the test material. If students select study partners that can distract them from schoolwork, estimators that do not take that sorting process into account can be downward biased. These results suggest that policies that strengthen collaboration between students within and outside the classroom can generate benefits that have the potential to generate positive social multipliers. All results are qualitatively robust to different choices of p and D_n; see Section D.2 below.

D.2 Supplementary Estimation Results

For robustness purposes, Tables 5-10 show the empirical estimation of model (6.1) with the kernel Tukey-Hanning, constant $C \in \{1.5, 1.6, 1.7\}$, and $p \in \{3, 4, 5\}$.

D.3 Assessing Assumptions

To validate the Assumption 2, we perform a Least Square (LS) regression that includes all variables in equation 6.1 and our proposed instruments. Namely, we estimate the following equation.

$$\begin{aligned} \texttt{math} &= \beta \mathbf{W}_n \texttt{math} + \texttt{personality}\, \gamma_p + \texttt{characteristics}\, \gamma_{ch} \\ &\quad + \mathbf{W}_n\, \texttt{characteristics}\, \boldsymbol{\delta} + \mathbf{W}_{n,0}\, \texttt{characteristics}\, \boldsymbol{\delta}_1 \\ &\quad + \mathbf{W}_{n,0}^2\, \texttt{characteristics}\, \boldsymbol{\delta}_2 + \text{error}, \end{aligned} \tag{D-1}$$

where `characteristics`, `personality`, and the adjacency matrices $\mathbf{W}_n$, $\mathbf{W}_{n,0}$ are defined above. The estimation results for equation (D-1) are shown in Table 11. They suggest no correlation between the output variable `math` and the vectors $\mathbf{W}_{n,0}$ `characteristics` and $\mathbf{W}_{n,0}^2$ `characteristics`, which are the proposed instruments. The estimated coefficients $\boldsymbol{\delta}_1$ and $\boldsymbol{\delta}_2$ are statistically insignificant[2], which allow us to use the exogenous variation embodied in $\mathbf{W}_{n,0}$ to identify the parameters of the linear model (6.1).

To validate Assumption 3, we run a series of LS regressions in which the outcome is the endogenous variable $\mathbf{W}_n$`math`. Namely, we estimate the following models.

[2]The estimated coefficient of $\mathbf{W}_{n,0}^2\ln(\text{Weight})$ is significant at 10%.

$$\mathbf{W}_n\texttt{math} = \mathbf{W}_{n,0}\ \texttt{characteristics}\ \boldsymbol{\delta} + \mathbf{W}_{n,0}\ \texttt{personality}\ \boldsymbol{\gamma} + \text{error}, \tag{D-2}$$

$$\mathbf{W}_n\texttt{math} = \mathbf{W}_{n,0}^2\ \texttt{characteristics}\ \boldsymbol{\delta} + \mathbf{W}_{n,0}^2\ \texttt{personality}\ \boldsymbol{\gamma} + \text{error}, \tag{D-3}$$

$$\begin{aligned}\mathbf{W}_n\texttt{math} = {} & \mathbf{W}_{n,0}\ \texttt{characteristics}\ \boldsymbol{\delta} + \mathbf{W}_{n,0}\ \texttt{personality}\ \boldsymbol{\gamma} \\ & + \mathbf{W}_{n,0}^2\ \texttt{characteristics}\ \boldsymbol{\delta} + \mathbf{W}_{n,0}^2\ \texttt{personality}\ \boldsymbol{\gamma} + \text{error}.\end{aligned} \tag{D-4}$$

The estimation results for these specifications are shown in Table 12. F-statistics suggest that our proposed instruments are relevant to describe the endogenous variable. Furthermore, we also perform LS regressions in which the dependent variable $\mathbf{W}_n\mathbf{x}$ is the average of characteristic s$\mathbf{x}$ between the study partners, which can be any of the characteristics mentioned above, for example height, weight, siblings help, parents help, commute to school by car or taxi; playing music; and whether the student is male.

$$\mathbf{W}_n\mathbf{x} = \mathbf{W}_{n,0}\ \texttt{characteristics}_{-\mathbf{x}}\ \boldsymbol{\delta} + \mathbf{W}_{n,0}\ \texttt{personality}\ \boldsymbol{\gamma} + \text{error}, \tag{D-5}$$

$$\mathbf{W}_n\mathbf{x} = \mathbf{W}_{n,0}^2\ \texttt{characteristics}_{-\mathbf{x}}\ \boldsymbol{\delta} + \mathbf{W}_{n,0}^2\ \texttt{personality}\ \boldsymbol{\gamma} + \text{error}, \tag{D-6}$$

$$\begin{aligned}\mathbf{W}_n\mathbf{x} = {} & \mathbf{W}_{n,0}\ \texttt{characteristics}_{-\mathbf{x}}\ \boldsymbol{\delta} + \mathbf{W}_{n,0}\ \texttt{personality}\ \boldsymbol{\gamma} + \text{error}, \\ & + \mathbf{W}_{n,0}^2\ \texttt{characteristics}_{-\mathbf{x}}\ \boldsymbol{\delta} + \mathbf{W}_{n,0}^2\ \texttt{personality}\ \boldsymbol{\gamma} + \text{error}.\end{aligned} \tag{D-7}$$

Here, $\texttt{characteristics}_{-\mathbf{x}}$ means that all characteristics have been included except $\mathbf{x}$, used in $\mathbf{W}_n\mathbf{x}$. Tables 13-19 show OLS estimates, and F-statistics also suggest that our proposed instruments are relevant to describe endogenous variables $\mathbf{W}_n\mathbf{x}$.

Finally, we present a network architecture that integrates students based on study partnerships and shared seating arrangements. Specifically, we establish a new set of connections, denoted $\mathbf{W}_{n,*}$, between students i and k following a defined rule: If students i and j mutually identified each other as study partners in the survey and students j and k reciprocated as seatmates, we establish an edge between i and k. This connection is formed when neither students i and k are study partners, nor are students j and k. Table 1 reports the summary statistics of this new network in the column named Extra. Based on this new set of connections, we rewrite the previous equations as follows.

$$
\begin{aligned}
\mathbf{W}_n\texttt{math} &= \mathbf{W}_{n,*}\ \texttt{characteristics}\ \boldsymbol{\delta} + \mathbf{W}_{n,*}\ \texttt{personality}\ \boldsymbol{\gamma} + \text{error}, && \text{(D-8)}\\
\mathbf{W}_n\texttt{math} &= \mathbf{W}_{n,*}^2\ \texttt{characteristics}\ \boldsymbol{\delta} + \mathbf{W}_{n,*}^2\ \texttt{personality}\ \boldsymbol{\gamma} + \text{error}, && \text{(D-9)}\\
\mathbf{W}_n\texttt{math} &= \mathbf{W}_{n,*}\ \texttt{characteristics}\ \boldsymbol{\delta} + \mathbf{W}_{n,*}\ \texttt{personality}\ \boldsymbol{\gamma} + \text{error}, && \text{(D-10)}\\
&\quad + \mathbf{W}_{n,*}^2\ \texttt{characteristics}\ \boldsymbol{\delta} + \mathbf{W}_{n,*}^2\ \texttt{personality}\ \boldsymbol{\gamma} + \text{error}.
\end{aligned}
$$

The LS estimates for all these three specifications are shown in Table 20. F-Statistics suggest that our proposed instruments are relevant to describe the endogenous variable.

Table 2: Summary Statistics

Variables	Scale	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
Student-related variables													
Math Test	[0,100]	61.88	13.56	33.57	92.33	61.44	13.12	31.00	92.00	68.38	14.01	24.35	100.00
Male		0.37	0.48	0.00	1.00	0.58	0.50	0.00	1.00	0.42	0.49	0.00	1.00
Height (cm)		156.50	7.48	139.00	176.00	157.41	7.80	133.00	175.00	161.05	9.18	100.00	208.30
Weight (kg)		46.56	9.15	27.00	72.00	46.86	11.27	28.00	99.00	48.33	10.64	26.30	130.00
Siblings Help		0.47	0.50	0.00	1.00	0.43	0.50	0.00	1.00	0.46	0.50	0.00	1.00
Parents Help		0.61	0.49	0.00	1.00	0.63	0.48	0.00	1.00	0.65	0.48	0.00	1.00
Music		0.53	0.58	0.00	2.00	0.66	0.62	0.00	3.00	0.85	0.56	0.00	3.00
Commute by car/taxi		0.68	0.47	0.00	1.00	0.53	0.50	0.00	1.00	0.58	0.49	0.00	1.00
Cognitive and Personality Tests													
Cognitive	[0,16]	7.80	1.66	3.00	12.00	7.94	1.78	4.00	12.00	8.92	1.88	2.00	14.00
Agreeableness	[9,40]	27.12	4.26	14.00	39.00	27.09	3.91	15.00	37.00	27.04	3.99	12.00	40.00
Conscientiousness	[9,45]	26.71	5.87	14.00	40.00	27.90	4.97	18.00	43.00	25.88	5.47	12.00	45.00
Extraversion	[8,40]	27.65	4.86	16.00	38.00	27.62	4.85	16.00	38.00	26.35	5.10	10.00	39.00
Neuroticism	[8,40]	22.31	5.83	9.00	36.00	21.95	5.36	8.00	35.00	23.45	5.57	9.00	38.00
Openness	[10,55]	37.45	5.45	24.00	50.00	36.65	5.09	19.00	51.00	35.26	5.48	18.00	51.00

Note: Descriptive statistics such as sample mean (Mean), standard deviation (SD), minimum (Min), maximum (Max) and sample size (n) are presented here for all variables and each school. Course grades, personality trait measures, and cognitive ability tests are scored on the scale indicated.

Table 3: Summary Network Statistics

	School 1			School 2			School 3		
Variables	Studymates	Seatmates	Extra	Studymates	Seatmates	Extra	Studymates	Seatmates	Extra
Number of nodes	133	133	133	171	171	171	564	564	564
Number of edges	171	175	454	228	275	748	819	799	2974
Density × 100	1.948	1.994	5.172	1.569	1.892	5.146	0.516	0.503	1.873
Average degree	2.571	2.632	6.827	2.667	3.216	8.749	2.904	2.833	10.546
Average clustering	0.213	0.068	0.209	0.153	0.066	0.228	0.129	0.063	0.160
Assortativity measure	0.190	0.266	0.187	0.039	0.194	0.256	0.177	0.177	0.067
Number of isolated node	15	7	3	19	3	5	69	13	5
Number of Subgraph	21	14	4	27	9	6	76	30	8
Transitivity	0.281	0.094	0.244	0.207	0.092	0.234	0.180	0.081	0.153

Note: The degree is multiplied by 100 to increase the scale.

Table 4: Estimations results, cont.

	OLS		G2SLS		GMM	
Variables	Coef.	SE	Coef.	SE	Coef.	SE
Music	-0.0196**	(0.0099)	-0.0031	(0.0130)	-0.0019	(0.0126)
Elder Siblings Help	0.0087	(0.0116)	-0.0030	(0.0126)	0.0001	(0.0138)
Younger Siblings Help	0.0435***	(0.0159)	0.0403**	(0.0201)	0.0382*	(0.0212)
Male × ln(Cognitive)	0.1275***	(0.0486)	0.1528**	(0.0680)	0.2008***	(0.0576)
Male × ln(Agreeableness)	-0.1318**	(0.0668)	-0.0318	(0.0622)	-0.0407	(0.0667)
Male × ln(Conscientiousness)	0.1216**	(0.0520)	0.1612**	(0.0639)	0.1582**	(0.0727)
Male × ln(Extraversion)	0.0266	(0.0404)	0.0565	(0.0559)	0.0434	(0.0641)
Male × ln(Neuroticism)	0.0441	(0.0518)	0.1033	(0.0630)	0.1003	(0.0658)
Male × ln(Openness)	-0.0157	(0.0566)	-0.0968	(0.0805)	-0.1089	(0.0945)
School 1, grade 7, class 1	0.1081**	(0.0516)	0.0417***	(0.0155)	0.0293*	(0.0176)
School 1, grade 7, class 2	0.0243	(0.0533)	-0.0682***	(0.0247)	-0.1288***	(0.0238)
School 1, grade 7, class 3	0.0854***	(0.0283)	0.0658***	(0.0106)	0.0779***	(0.0111)
School 1, grade 7, class 4	0.1583***	(0.0525)	0.0889***	(0.0148)	0.1177***	(0.0132)
School 1, grade 7, class 5	0.1897***	(0.0493)	0.1225***	(0.0154)	0.1394***	(0.0124)
School 2, grade 7, class 1	0.0938***	(0.0275)	0.0204	(0.0125)	0.0162	(0.0135)
School 2, grade 7, class 2	0.0516	(0.0326)	-0.0120	(0.0132)	-0.0172	(0.0159)
School 2, grade 7, class 3	0.1350***	(0.0428)	0.0538***	(0.0153)	0.0649***	(0.0155)
School 2, grade 7, class 4	0.0720**	(0.0316)	0.0472***	(0.0141)	0.0390**	(0.0161)
School 2, grade 7, class 5	0.1312***	(0.0449)	0.1126***	(0.0180)	0.1480***	(0.0164)
School 3, grade 7, class 1	0.0948**	(0.0433)	0.1207***	(0.0159)	0.1541***	(0.0131)
School 3, grade 7, class 2	0.0983**	(0.0434)	0.1335***	(0.0165)	0.1755***	(0.0117)
School 3, grade 7, class 3	0.0900**	(0.0438)	0.1488***	(0.0172)	0.1916***	(0.0170)
School 3, grade 7, class 4	0.0719*	(0.0385)	0.1034***	(0.0153)	0.1294***	(0.0129)
School 3, grade 7, class 5	0.0612	(0.0453)	0.0944***	(0.0202)	0.1384***	(0.0151)
School 3, grade 8, class 1	0.1364***	(0.0379)	0.1232***	(0.0144)	0.1547***	(0.0096)
School 3, grade 8, class 2	0.1223**	(0.0520)	0.1482***	(0.0235)	0.2016***	(0.0162)
School 3, grade 8, class 3	0.0982***	(0.0334)	0.1308***	(0.0150)	0.1619***	(0.0116)
School 3, grade 8, class 4	0.1069**	(0.0484)	0.1493***	(0.0219)	0.1997***	(0.0172)
School 3, grade 8, class 5	0.0770*	(0.0411)	0.1087***	(0.0175)	0.1545***	(0.0115)
School 3, grade 9, class 1	0.0409*	(0.0246)	0.0782***	(0.0075)	0.0693***	(0.0072)
School 3, grade 9, class 2	0.0384	(0.0263)	0.0557***	(0.0066)	0.0575***	(0.0090)
School 3, grade 9, class 3	0.0374	(0.0295)	0.0543***	(0.0082)	0.0518***	(0.0080)
School 3, grade 9, class 4	0.0382	(0.0291)	0.0798***	(0.0072)	0.0819***	(0.0084)
Constant	-4.5872*	(2.5808)	2.9464***	(0.9048)	3.8210***	(1.2635)

Note: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 5: Estimations results with $p = 3$

Variables	$C = 1.5$	$C = 1.6$	$C = 1.7$
Peer effect			
ln(Math Test)	0.7002*	0.8889***	0.8919***
	(0.3737)	(0.3157)	(0.3069)
Contextual effects			
Male	-0.2958	-0.2395*	-0.2403*
	(0.1836)	(0.1436)	(0.1417)
ln(Height)	3.8548*	2.2241**	2.2012**
	(2.0045)	(1.0261)	(1.0108)
ln(Weight)	0.0245	-0.0452	-0.0558
	(0.4489)	(0.3267)	(0.3123)
Siblings Help	-0.0049	0.0674	0.0630
	(0.1390)	(0.0833)	(0.0812)
Parents Help	-0.0303	-0.1154	-0.1114
	(0.1479)	(0.0841)	(0.0823)
Commute by Car/Taxi	0.0857	0.1395	0.1515
	(0.1307)	(0.1126)	(0.1110)
Music	0.0575	0.0842	0.0782
	(0.0989)	(0.0841)	(0.0822)
†			
ln(Cognitive)	0.1119**	0.1318***	0.1322***
	(0.0494)	(0.0437)	(0.0421)
ln(Agreeableness)	-0.0953	-0.1254**	-0.1204*
	(0.0720)	(0.0632)	(0.0618)
ln(Conscientiousness)	0.0738	0.0649	0.0647
	(0.0740)	(0.0718)	(0.0713)
ln(Extraversion)	-0.1552**	-0.1290**	-0.1259**
	(0.0768)	(0.0614)	(0.0608)
ln(Neuroticism)	-0.0003	-0.0333	-0.0331
	(0.0447)	(0.0396)	(0.0389)
ln(Openness)	0.0229	0.0115	0.0068
	(0.0741)	(0.0654)	(0.0645)
Direct effects			
Male	-0.2902	-0.6800	-0.6580
	(0.8509)	(0.8366)	(0.8331)
ln(Height)	-1.3506**	-0.9101***	-0.9102***
	(0.5345)	(0.3046)	(0.2987)
ln(Weight)	-0.0692	-0.0238	-0.0187
	(0.0966)	(0.0759)	(0.0725)
Siblings Help	-0.0407	-0.0640**	-0.0627**
	(0.0444)	(0.0282)	(0.0273)
Parents Help	0.0229	0.0378*	0.0371*
	(0.0295)	(0.0203)	(0.0200)
Commute by Car/Taxi	-0.0229	-0.0315*	-0.0335*
	(0.0190)	(0.0176)	(0.0171)
Degree	0.0258***	0.0209***	0.0208***
	(0.0067)	(0.0058)	(0.0057)
Isolate Students	0.3446	0.4858	0.4755
	(0.4991)	(0.4822)	(0.4766)
n	868	868	868
Adjusted R^2	0.2100	0.2617	0.2615
RMSE	0.2205	0.2092	0.2094

Note: (i) * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$; (ii) Standard errors are in parentheses. (iii) † These regressors are measured as the deviation of students' personality from their peers' average.

Table 6: Estimations results with $p = 3$, cont.

	$C = 1.5$		$C = 1.6$		$C = 1.7$	
Variables	Coef.	SE	Coef.	SE	Coef.	SE
Music	-0.0164	(0.0189)	-0.0115	(0.0163)	-0.0102	(0.0158)
Elder Siblings Help	0.0171	(0.0207)	0.0086	(0.0173)	0.0073	(0.0169)
Younger Siblings Help	0.0357	(0.0296)	0.0470**	(0.0218)	0.0470**	(0.0216)
Male × ln(Cognitive)	0.0823	(0.0951)	0.0802	(0.0902)	0.0788	(0.0875)
Male × ln(Agreeableness)	-0.0804	(0.1019)	-0.0349	(0.1017)	-0.0354	(0.0997)
Male × ln(Conscientiousness)	0.1095	(0.0821)	0.1334*	(0.0751)	0.1324*	(0.0736)
Male × ln(Extraversion)	-0.0066	(0.0830)	0.0313	(0.0632)	0.0286	(0.0626)
Male × ln(Neuroticism)	0.0384	(0.0846)	0.0700	(0.0777)	0.0660	(0.0776)
Male × ln(Openness)	0.0417	(0.0925)	0.0239	(0.0821)	0.0261	(0.0798)
School 1, grade 7, class 1	0.1649**	(0.0771)	0.1201**	(0.0604)	0.1136*	(0.0592)
School 1, grade 7, class 2	0.0582	(0.1200)	0.0671	(0.1028)	0.0607	(0.0998)
School 1, grade 7, class 3	0.1181**	(0.0460)	0.0711*	(0.0367)	0.0687*	(0.0354)
School 1, grade 7, class 4	0.2019**	(0.0891)	0.1097	(0.0687)	0.1033	(0.0671)
School 1, grade 7, class 5	0.2346***	(0.0808)	0.1595**	(0.0639)	0.1525**	(0.0624)
School 2, grade 7, class 1	0.1464**	(0.0568)	0.1190***	(0.0414)	0.1146***	(0.0392)
School 2, grade 7, class 2	0.1105	(0.0674)	0.0621	(0.0532)	0.0609	(0.0509)
School 2, grade 7, class 3	0.1861***	(0.0704)	0.1174**	(0.0564)	0.1107**	(0.0544)
School 2, grade 7, class 4	0.1138*	(0.0600)	0.1154**	(0.0463)	0.1113**	(0.0444)
School 2, grade 7, class 5	0.1701**	(0.0767)	0.1049	(0.0644)	0.1016	(0.0624)
School 3, grade 7, class 1	0.1646**	(0.0769)	0.0778	(0.0569)	0.0737	(0.0558)
School 3, grade 7, class 2	0.1504*	(0.0808)	0.0727	(0.0623)	0.0688	(0.0611)
School 3, grade 7, class 3	0.1408*	(0.0744)	0.0634	(0.0600)	0.0609	(0.0587)
School 3, grade 7, class 4	0.1212*	(0.0631)	0.0701	(0.0488)	0.0665	(0.0475)
School 3, grade 7, class 5	0.0904	(0.0741)	0.0268	(0.0611)	0.0242	(0.0600)
School 3, grade 8, class 1	0.1560**	(0.0616)	0.1077**	(0.0508)	0.1055**	(0.0496)
School 3, grade 8, class 2	0.1342	(0.0906)	0.0715	(0.0770)	0.0690	(0.0751)
School 3, grade 8, class 3	0.0943	(0.0590)	0.0611	(0.0492)	0.0586	(0.0480)
School 3, grade 8, class 4	0.1327	(0.0845)	0.0601	(0.0714)	0.0568	(0.0700)
School 3, grade 8, class 5	0.0750	(0.0640)	0.0438	(0.0592)	0.0407	(0.0574)
School 3, grade 9, class 1	0.0446	(0.0355)	0.0515*	(0.0301)	0.0486	(0.0298)
School 3, grade 9, class 2	0.0319	(0.0427)	0.0269	(0.0338)	0.0238	(0.0331)
School 3, grade 9, class 3	0.0239	(0.0554)	0.0308	(0.0394)	0.0270	(0.0383)
School 3, grade 9, class 4	0.0428	(0.0380)	0.0500	(0.0336)	0.0488	(0.0334)
Constant	-11.5128*	(6.3535)	-6.1593*	(3.3041)	-6.0328*	(3.2568)

Note: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 7: Estimations results with $p = 4$

Variables	$C = 1.5$	$C = 1.6$	$C = 1.7$
Peer effect			
ln(Math Test)	0.6729***	0.6762***	0.6791***
	(0.2236)	(0.2159)	(0.2090)
Contextual effects			
Male	-0.2057*	-0.2111*	-0.2161**
	(0.1115)	(0.1080)	(0.1046)
ln(Height)	2.2455***	2.2006***	2.1608***
	(0.8568)	(0.8471)	(0.8359)
ln(Weight)	0.0443	0.0460	0.0486
	(0.2703)	(0.2653)	(0.2607)
Siblings Help	0.0387	0.0399	0.0419
	(0.0567)	(0.0546)	(0.0525)
Parents Help	-0.0205	-0.0133	-0.0072
	(0.0658)	(0.0639)	(0.0620)
Commute by Car/Taxi	0.0649	0.0733	0.0799
	(0.0688)	(0.0669)	(0.0653)
Music	0.1161*	0.1143**	0.1131**
	(0.0600)	(0.0582)	(0.0565)
†			
ln(Cognitive)	0.1098***	0.1107***	0.1118***
	(0.0378)	(0.0361)	(0.0345)
ln(Agreeableness)	-0.0963*	-0.0902*	-0.0844*
	(0.0500)	(0.0483)	(0.0466)
ln(Conscientiousness)	0.0915*	0.0930*	0.0946*
	(0.0517)	(0.0501)	(0.0487)
ln(Extraversion)	-0.1227***	-0.1206**	-0.1185**
	(0.0474)	(0.0469)	(0.0462)
ln(Neuroticism)	-0.0150	-0.0136	-0.0122
	(0.0293)	(0.0284)	(0.0275)
ln(Openness)	0.0221	0.0179	0.0138
	(0.0493)	(0.0475)	(0.0458)
Direct effects			
Male	-0.4717	-0.4442	-0.4138
	(0.6411)	(0.6298)	(0.6167)
ln(Height)	-0.9817***	-0.9767***	-0.9699***
	(0.2697)	(0.2625)	(0.2553)
ln(Weight)	-0.0685	-0.0668	-0.0656
	(0.0675)	(0.0654)	(0.0637)
Siblings Help	-0.0347	-0.0351	-0.0360*
	(0.0235)	(0.0227)	(0.0218)
Parents Help	0.0210	0.0186	0.0168
	(0.0168)	(0.0163)	(0.0158)
Commute by Car/Taxi	-0.0160	-0.0175	-0.0188
	(0.0156)	(0.0153)	(0.0150)
Degree	0.0248***	0.0247***	0.0247***
	(0.0044)	(0.0042)	(0.0041)
Isolate Students	0.2403	0.2186	0.1963
	(0.3495)	(0.3417)	(0.3327)
n	868	868	868
Adjusted R^2	0.2800	0.2813	0.2821
RMSE	0.1980	0.1979	0.1980

Note: (i) * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$; (ii) Standard errors are in parentheses. (iii) † These regressors are measured as the deviation of students' personality from their peers' average.

Table 8: Estimations results with $p = 4$, cont.

	$C = 1.5$		$C = 1.6$		$C = 1.7$	
Variables	Coef.	SE	Coef.	SE	Coef.	SE
Music	-0.0232*	(0.0119)	-0.0225*	(0.0115)	-0.0219*	(0.0112)
Elder Siblings Help	0.0052	(0.0129)	0.0055	(0.0125)	0.0060	(0.0121)
Younger Siblings Help	0.0372*	(0.0192)	0.0380**	(0.0190)	0.0389**	(0.0188)
Male × ln(Cognitive)	0.1055	(0.0675)	0.1049	(0.0640)	0.1044*	(0.0609)
Male × ln(Agreeableness)	-0.0848	(0.0879)	-0.0866	(0.0857)	-0.0885	(0.0836)
Male × ln(Conscientiousness)	0.1311**	(0.0634)	0.1297**	(0.0614)	0.1276**	(0.0599)
Male × ln(Extraversion)	0.0488	(0.0488)	0.0473	(0.0461)	0.0460	(0.0437)
Male × ln(Neuroticism)	0.0720	(0.0620)	0.0703	(0.0610)	0.0682	(0.0600)
Male × ln(Openness)	-0.0239	(0.0706)	-0.0242	(0.0672)	-0.0246	(0.0641)
School 1, grade 7, class 1	0.1284**	(0.0519)	0.1232**	(0.0510)	0.1189**	(0.0502)
School 1, grade 7, class 2	0.0504	(0.0677)	0.0469	(0.0656)	0.0439	(0.0636)
School 1, grade 7, class 3	0.0897***	(0.0324)	0.0879***	(0.0312)	0.0867***	(0.0301)
School 1, grade 7, class 4	0.1606***	(0.0563)	0.1568***	(0.0555)	0.1540***	(0.0547)
School 1, grade 7, class 5	0.1954***	(0.0528)	0.1902***	(0.0518)	0.1859***	(0.0507)
School 2, grade 7, class 1	0.0985***	(0.0343)	0.0960***	(0.0320)	0.0935***	(0.0299)
School 2, grade 7, class 2	0.0584	(0.0450)	0.0571	(0.0430)	0.0570	(0.0412)
School 2, grade 7, class 3	0.1376***	(0.0457)	0.1335***	(0.0443)	0.1304***	(0.0429)
School 2, grade 7, class 4	0.0852**	(0.0343)	0.0823**	(0.0328)	0.0802**	(0.0312)
School 2, grade 7, class 5	0.1242**	(0.0494)	0.1226**	(0.0479)	0.1219***	(0.0465)
School 3, grade 7, class 1	0.1041**	(0.0477)	0.0995**	(0.0468)	0.0955**	(0.0460)
School 3, grade 7, class 2	0.1009**	(0.0498)	0.0988**	(0.0486)	0.0968**	(0.0476)
School 3, grade 7, class 3	0.0940*	(0.0497)	0.0909*	(0.0483)	0.0882*	(0.0470)
School 3, grade 7, class 4	0.0791*	(0.0431)	0.0757*	(0.0416)	0.0729*	(0.0401)
School 3, grade 7, class 5	0.0560	(0.0507)	0.0542	(0.0497)	0.0527	(0.0488)
School 3, grade 8, class 1	0.1302***	(0.0396)	0.1290***	(0.0383)	0.1280***	(0.0373)
School 3, grade 8, class 2	0.1121*	(0.0590)	0.1101*	(0.0572)	0.1085*	(0.0555)
School 3, grade 8, class 3	0.0903**	(0.0382)	0.0879**	(0.0371)	0.0858**	(0.0361)
School 3, grade 8, class 4	0.1058*	(0.0550)	0.1025*	(0.0536)	0.0996*	(0.0523)
School 3, grade 8, class 5	0.0644	(0.0465)	0.0635	(0.0450)	0.0629	(0.0437)
School 3, grade 9, class 1	0.0475*	(0.0256)	0.0442*	(0.0256)	0.0413	(0.0253)
School 3, grade 9, class 2	0.0424	(0.0297)	0.0390	(0.0292)	0.0359	(0.0289)
School 3, grade 9, class 3	0.0483	(0.0342)	0.0434	(0.0332)	0.0394	(0.0323)
School 3, grade 9, class 4	0.0358	(0.0309)	0.0339	(0.0307)	0.0319	(0.0302)
Constant	-5.2394*	(2.7824)	-5.0676*	(2.7405)	-4.9306*	(2.6995)

Note: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 9: Estimations results with $p = 5$

Variables	$C = 1.5$	$C = 1.6$	$C = 1.7$
Peer effect			
ln(Math Test)	0.5943***	0.5987***	0.6065***
	(0.1923)	(0.1836)	(0.1755)
Contextual effects			
Male	-0.2494***	-0.2466***	-0.2455***
	(0.0949)	(0.0909)	(0.0873)
ln(Height)	2.3137***	2.2570***	2.1682***
	(0.7917)	(0.7660)	(0.7400)
ln(Weight)	0.0491	0.0399	0.0329
	(0.2123)	(0.2031)	(0.1942)
Siblings Help	0.0263	0.0274	0.0294
	(0.0521)	(0.0505)	(0.0491)
Parents Help	-0.0326	-0.0207	-0.0106
	(0.0590)	(0.0554)	(0.0517)
Commute by Car/Taxi	0.1147*	0.1205*	0.1255**
	(0.0683)	(0.0659)	(0.0636)
Music	0.1243**	0.1206**	0.1187**
	(0.0531)	(0.0505)	(0.0480)
†			
ln(Cognitive)	0.1093***	0.1087***	0.1086***
	(0.0317)	(0.0300)	(0.0286)
ln(Agreeableness)	-0.0866*	-0.0787	-0.0712
	(0.0501)	(0.0479)	(0.0457)
ln(Conscientiousness)	0.0752*	0.0774*	0.0799**
	(0.0435)	(0.0418)	(0.0406)
ln(Extraversion)	-0.1229***	-0.1202***	-0.1164***
	(0.0420)	(0.0412)	(0.0405)
ln(Neuroticism)	-0.0132	-0.0124	-0.0113
	(0.0278)	(0.0266)	(0.0257)
ln(Openness)	0.0353	0.0311	0.0272
	(0.0456)	(0.0435)	(0.0414)
Direct effects			
Male	-0.2089	-0.1901	-0.1648
	(0.5776)	(0.5609)	(0.5417)
ln(Height)	-1.0302***	-1.0141***	-0.9904***
	(0.2436)	(0.2343)	(0.2248)
ln(Weight)	-0.0578	-0.0537	-0.0494
	(0.0516)	(0.0483)	(0.0453)
Siblings Help	-0.0394*	-0.0391*	-0.0395*
	(0.0225)	(0.0214)	(0.0204)
Parents Help	0.0224	0.0194	0.0167
	(0.0155)	(0.0147)	(0.0139)
Commute by Car/Taxi	-0.0216	-0.0234*	-0.0251*
	(0.0145)	(0.0140)	(0.0136)
Degree	0.0251***	0.0250***	0.0248***
	(0.0037)	(0.0036)	(0.0034)
Isolate Students	0.2024	0.1752	0.1425
	(0.3118)	(0.3022)	(0.2935)
n	868	868	868
Adjusted R^2	0.2593	0.2636	0.2675
RMSE	0.2020	0.2010	0.2002

Note: (i) * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$; (ii) Standard errors are in parentheses. (iii) † These regressors are measured as the deviation of students' personality from their peers' average.

Table 10: Estimations results with $p = 5$, cont.

	$C = 1.5$		$C = 1.6$		$C = 1.7$	
Variables	Coef.	SE	Coef.	SE	Coef.	SE
Music	-0.0204*	(0.0110)	-0.0199*	(0.0105)	-0.0196**	(0.0099)
Elder Siblings Help	0.0088	(0.0125)	0.0087	(0.0121)	0.0087	(0.0116)
Younger Siblings Help	0.0410**	(0.0169)	0.0421**	(0.0164)	0.0435***	(0.0159)
Male × ln(Cognitive)	0.1349**	(0.0538)	0.1313**	(0.0510)	0.1275***	(0.0486)
Male × ln(Agreeableness)	-0.1317*	(0.0730)	-0.1321*	(0.0702)	-0.1318**	(0.0668)
Male × ln(Conscientiousness)	0.1256**	(0.0563)	0.1240**	(0.0539)	0.1216**	(0.0520)
Male × ln(Extraversion)	0.0337	(0.0472)	0.0298	(0.0438)	0.0266	(0.0404)
Male × ln(Neuroticism)	0.0425	(0.0538)	0.0435	(0.0527)	0.0441	(0.0518)
Male × ln(Openness)	-0.0163	(0.0633)	-0.0154	(0.0598)	-0.0157	(0.0566)
School 1, grade 7, class 1	0.1198**	(0.0543)	0.1142**	(0.0530)	0.1081**	(0.0516)
School 1, grade 7, class 2	0.0304	(0.0583)	0.0268	(0.0557)	0.0243	(0.0533)
School 1, grade 7, class 3	0.0877***	(0.0309)	0.0869***	(0.0296)	0.0854***	(0.0283)
School 1, grade 7, class 4	0.1716***	(0.0540)	0.1652***	(0.0533)	0.1583***	(0.0525)
School 1, grade 7, class 5	0.2042***	(0.0521)	0.1973***	(0.0508)	0.1897***	(0.0493)
School 2, grade 7, class 1	0.1028***	(0.0324)	0.0985***	(0.0299)	0.0938***	(0.0275)
School 2, grade 7, class 2	0.0573	(0.0370)	0.0541	(0.0347)	0.0516	(0.0326)
School 2, grade 7, class 3	0.1482***	(0.0456)	0.1418***	(0.0443)	0.1350***	(0.0428)
School 2, grade 7, class 4	0.0801**	(0.0356)	0.0759**	(0.0337)	0.0720**	(0.0316)
School 2, grade 7, class 5	0.1387***	(0.0478)	0.1352***	(0.0463)	0.1312***	(0.0449)
School 3, grade 7, class 1	0.1047**	(0.0460)	0.1005**	(0.0446)	0.0948**	(0.0433)
School 3, grade 7, class 2	0.1046**	(0.0458)	0.1022**	(0.0446)	0.0983**	(0.0434)
School 3, grade 7, class 3	0.0969**	(0.0471)	0.0940**	(0.0454)	0.0900**	(0.0438)
School 3, grade 7, class 4	0.0816**	(0.0409)	0.0770*	(0.0397)	0.0719*	(0.0385)
School 3, grade 7, class 5	0.0682	(0.0468)	0.0652	(0.0461)	0.0612	(0.0453)
School 3, grade 8, class 1	0.1428***	(0.0403)	0.1398***	(0.0391)	0.1364***	(0.0379)
School 3, grade 8, class 2	0.1314**	(0.0551)	0.1271**	(0.0535)	0.1223**	(0.0520)
School 3, grade 8, class 3	0.1049***	(0.0352)	0.1016***	(0.0343)	0.0982***	(0.0334)
School 3, grade 8, class 4	0.1167**	(0.0509)	0.1124**	(0.0496)	0.1069**	(0.0484)
School 3, grade 8, class 5	0.0818*	(0.0440)	0.0795*	(0.0425)	0.0770*	(0.0411)
School 3, grade 9, class 1	0.0450*	(0.0254)	0.0428*	(0.0251)	0.0409*	(0.0246)
School 3, grade 9, class 2	0.0415	(0.0279)	0.0401	(0.0271)	0.0384	(0.0263)
School 3, grade 9, class 3	0.0476	(0.0324)	0.0421	(0.0310)	0.0374	(0.0295)
School 3, grade 9, class 4	0.0430	(0.0306)	0.0407	(0.0300)	0.0382	(0.0291)
Constant	-5.0956*	(2.7565)	-4.8916*	(2.6662)	-4.5872*	(2.5808)

Note: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 11: LS Estimation results of the model $y = \beta \mathbf{W}\mathrm{y} + \boldsymbol{\delta}\mathrm{X} + \boldsymbol{\gamma}\mathbf{W}\mathrm{X} + \boldsymbol{\theta}_1 \mathbf{W}_0 \mathrm{X} + \boldsymbol{\theta}_2 \mathbf{W}_0^2 \mathrm{X}$

Variables	Coef.	SE	Variables	Coef.	SE
ln(Math Test)	0.2204***	(0.0784)	$\mathbf{W}_{n,0}^2$ × Male	-0.0512	(0.0466)
Male	-0.3938	(0.4050)	$\mathbf{W}_{n,0}^2$ × ln(Height)	-0.3393	(0.2333)
ln(Height)	-0.2541*	(0.1477)	$\mathbf{W}_{n,0}^2$ × ln(Weight)	0.1710*	(0.0903)
ln(Weight)	-0.0669*	(0.0392)	$\mathbf{W}_{n,0}^2$ × Siblings Help	0.0571	(0.0470)
Siblings Help	-0.0262	(0.0223)	$\mathbf{W}_{n,0}^2$ × Parents Help	-0.0117	(0.0557)
Parents Help	0.0290	(0.0248)	$\mathbf{W}_{n,0}^2$ × Commute by Car/Taxi	0.0111	(0.0508)
Commute by Car/Taxi	-0.0104	(0.0200)	$\mathbf{W}_{n,0}^2$ × Music	0.0251	(0.0363)
Music	-0.0091	(0.0132)	School 1, grade 7, class 1	0.0105	(0.0232)
Degree	0.0281***	(0.0040)	School 1, grade 7, class 2	-0.1337***	(0.0251)
Isolate Students	1.2398	(1.1350)	School 1, grade 7, class 3	0.0768***	(0.0127)
Elder Siblings Help	-0.0002	(0.0140)	School 1, grade 7, class 4	0.1075***	(0.0206)
Younger Siblings Help	0.0364*	(0.0191)	School 1, grade 7, class 5	0.1462***	(0.0167)
Male × ln(Cognitive)	0.2563***	(0.0452)	School 2, grade 7, class 1	0.0233	(0.0150)
Male × ln(Agreeableness)	-0.1395**	(0.0588)	School 2, grade 7, class 2	-0.0049	(0.0244)
Male × ln(Conscientiousness)	0.2086***	(0.0624)	School 2, grade 7, class 3	0.0742***	(0.0175)
Male × ln(Extraversion)	-0.0559	(0.0543)	School 2, grade 7, class 4	0.0487***	(0.0177)
Male × ln(Neuroticism)	0.0632	(0.0537)	School 2, grade 7, class 5	0.1526***	(0.0227)
Male × ln(Openness)	-0.0795	(0.0802)	School 3, grade 7, class 1	0.1593***	(0.0188)
$\mathbf{W}_n$ × Male	-0.0330	(0.0299)	School 3, grade 7, class 2	0.1710***	(0.0187)
$\mathbf{W}_n$ × ln(Height)	-0.0563	(0.2103)	School 3, grade 7, class 3	0.1997***	(0.0270)
$\mathbf{W}_n$ × ln(Weight)	0.1383**	(0.0673)	School 3, grade 7, class 4	0.1355***	(0.0249)
$\mathbf{W}_n$ × Siblings Help	0.0321	(0.0259)	School 3, grade 7, class 5	0.1486***	(0.0184)
$\mathbf{W}_n$ × Parents Help	0.0438*	(0.0264)	School 3, grade 8, class 1	0.1516***	(0.0167)
$\mathbf{W}_n$ × Commute by Car/Taxi	0.0334	(0.0264)	School 3, grade 8, class 2	0.2023***	(0.0183)
$\mathbf{W}_n$ × Music	-0.0236	(0.0201)	School 3, grade 8, class 3	0.1563***	(0.0129)
$\mathbf{W}_{n,0}$ × Male	-0.0248	(0.0149)	School 3, grade 8, class 4	0.1964***	(0.0215)
$\mathbf{W}_{n,0}$ × ln(Height)	0.2446	(0.2356)	School 3, grade 8, class 5	0.1676***	(0.0112)
$\mathbf{W}_{n,0}$ × ln(Weight)	-0.0355	(0.0528)	School 3, grade 9, class 1	0.0651***	(0.0125)
$\mathbf{W}_{n,0}$ × Siblings Help	-0.0151	(0.0213)	School 3, grade 9, class 2	0.0449***	(0.0067)
$\mathbf{W}_{n,0}$ × Parents Help	-0.0106	(0.0206)	School 3, grade 9, class 3	0.0508***	(0.0085)
$\mathbf{W}_{n,0}$ × Commute by Car/Taxi	-0.0118	(0.0245)	School 3, grade 9, class 4	0.0668***	(0.0159)
$\mathbf{W}_{n,0}$ × Music	-0.0036	(0.0169)	Constant	4.2484***	(1.2059)
n	868				
F-Statistics	12.9000				
p-value	0.0000				

Note: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 12: LS Estimations using $\mathbf{W}y$ as dependent variable

	(D-2)		(D-3)		(D-4)	
Variables	Coef.	SE	Coef.	SE	Coef.	SE
$\mathbf{W}_{n,0} \times$ Male	0.1277	(0.1521)			0.2244	(0.1505)
$\mathbf{W}_{n,0} \times$ ln(Height)	1.1821**	(0.5783)			1.6369	(2.0158)
$\mathbf{W}_{n,0} \times$ ln(Weight)	-0.5142	(0.3457)			-0.2787	(0.4065)
$\mathbf{W}_{n,0} \times$ Siblings Help	0.1925	(0.1588)			0.1315	(0.1354)
$\mathbf{W}_{n,0} \times$ Parents Help	0.1572	(0.1427)			0.2092	(0.1405)
$\mathbf{W}_{n,0} \times$ Commute by Car/Taxi	0.0588	(0.1407)			0.0455	(0.1457)
$\mathbf{W}_{n,0} \times$ Music	0.2277**	(0.1077)			0.0790	(0.1280)
$\mathbf{W}_{n,0} \times$ ln(Cognitive)	-0.1062	(0.1635)			0.3246	(0.2969)
$\mathbf{W}_{n,0} \times$ ln(Agreeableness)	0.1843	(0.2616)			-0.0557	(0.4539)
$\mathbf{W}_{n,0} \times$ ln(Conscientiousness)	0.6115***	(0.2035)			0.0661	(0.3383)
$\mathbf{W}_{n,0} \times$ ln(Extraversion)	0.1699	(0.1471)			0.2083	(0.2455)
$\mathbf{W}_{n,0} \times$ ln(Neuroticism)	0.0704	(0.1952)			-0.0816	(0.2820)
$\mathbf{W}_{n,0} \times$ ln(Openness)	0.2039	(0.2609)			0.2749	(0.3080)
$\mathbf{W}^2_{n,0} \times$ Male			-0.2862	(0.2443)	-0.3681	(0.2556)
$\mathbf{W}^2_{n,0} \times$ ln(Height)			1.1908	(0.8774)	-0.5011	(1.9841)
$\mathbf{W}^2_{n,0} \times$ ln(Weight)			-0.2088	(0.6695)	0.0118	(0.7225)
$\mathbf{W}^2_{n,0} \times$ Siblings Help			0.5321***	(0.1883)	0.5106***	(0.1875)
$\mathbf{W}^2_{n,0} \times$ Parents Help			0.0411	(0.2204)	0.0127	(0.2210)
$\mathbf{W}^2_{n,0} \times$ Commute by Car/Taxi			-0.1939	(0.2112)	-0.1865	(0.2059)
$\mathbf{W}^2_{n,0} \times$ Music			0.4272***	(0.1628)	0.4009**	(0.1825)
$\mathbf{W}^2_{n,0} \times$ ln(Cognitive)			-0.3517	(0.2263)	-0.7223	(0.4509)
$\mathbf{W}^2_{n,0} \times$ ln(Agreeableness)			0.3012	(0.4596)	0.4606	(0.7711)
$\mathbf{W}^2_{n,0} \times$ ln(Conscientiousness)			1.1246***	(0.2488)	1.0246**	(0.4803)
$\mathbf{W}^2_{n,0} \times$ ln(Extraversion)			0.1388	(0.3130)	-0.1316	(0.5399)
$\mathbf{W}^2_{n,0} \times$ ln(Neuroticism)			0.0915	(0.3264)	0.1898	(0.5169)
$\mathbf{W}^2_{n,0} \times$ ln(Openness)			0.1003	(0.4454)	-0.2281	(0.5347)
Constant	-0.7597	(1.8614)	-1.8886	(2.8393)	-1.6844	(2.9717)
n	868		868		868	
F-Statistics	4.1245		9.2074		12.3000	
p-value	0.0013		0.0000		0.0000	

Note: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 13: LS Estimations using $\mathbf{W} \times$ Male as dependent variable

	(D-5)		(D-6)		(D-7)	
Variables	Coef.	SE	Coef.	SE	Coef.	SE
$\mathbf{W}_{n,0} \times$ ln(Height)	-0.1938*	(0.1152)			-0.8460*	(0.4750)
$\mathbf{W}_{n,0} \times$ ln(Weight)	0.1995**	(0.0950)			0.1754*	(0.0922)
$\mathbf{W}_{n,0} \times$ Siblings Help	0.0570	(0.0516)			0.0652	(0.0486)
$\mathbf{W}_{n,0} \times$ Parents Help	-0.0086	(0.0494)			-0.0142	(0.0496)
$\mathbf{W}_{n,0} \times$ Commute by Car/Taxi	-0.0495	(0.0511)			-0.0470	(0.0510)
$\mathbf{W}_{n,0} \times$ Music	-0.0159	(0.0496)			0.0092	(0.0491)
$\mathbf{W}_{n,0} \times$ ln(Cognitive)	0.2337***	(0.0448)			0.1023	(0.0700)
$\mathbf{W}_{n,0} \times$ ln(Agreeableness)	-0.1969***	(0.0622)			0.1203	(0.1030)
$\mathbf{W}_{n,0} \times$ ln(Conscientiousness)	0.1031	(0.0627)			0.0781	(0.1071)
$\mathbf{W}_{n,0} \times$ ln(Extraversion)	0.0437	(0.0367)			0.0723	(0.0714)
$\mathbf{W}_{n,0} \times$ ln(Neuroticism)	-0.1813***	(0.0502)			-0.0531	(0.0784)
$\mathbf{W}_{n,0} \times$ ln(Openness)	-0.0243	(0.0876)			-0.0344	(0.0896)
$\mathbf{W}^2_{n,0} \times$ ln(Height)			-0.3383*	(0.1797)	0.3152	(0.4610)
$\mathbf{W}^2_{n,0} \times$ ln(Weight)			0.2465	(0.1653)	0.2330	(0.1827)
$\mathbf{W}^2_{n,0} \times$ Siblings Help			0.0158	(0.0582)	0.0296	(0.0621)
$\mathbf{W}^2_{n,0} \times$ Parents Help			0.0624	(0.0618)	0.0802	(0.0579)
$\mathbf{W}^2_{n,0} \times$ Commute by Car/Taxi			-0.0127	(0.0620)	-0.0032	(0.0651)
$\mathbf{W}^2_{n,0} \times$ Music			-0.0940*	(0.0505)	-0.0906*	(0.0483)
$\mathbf{W}^2_{n,0} \times$ ln(Cognitive)			0.4096***	(0.0722)	0.2789**	(0.1191)
$\mathbf{W}^2_{n,0} \times$ ln(Agreeableness)			-0.5443***	(0.1188)	-0.7011***	(0.1642)
$\mathbf{W}^2_{n,0} \times$ ln(Conscientiousness)			0.2130**	(0.0943)	0.1059	(0.1780)
$\mathbf{W}^2_{n,0} \times$ ln(Extraversion)			0.0224	(0.0805)	-0.0703	(0.1309)
$\mathbf{W}^2_{n,0} \times$ ln(Neuroticism)			-0.2919***	(0.0801)	-0.2427*	(0.1313)
$\mathbf{W}^2_{n,0} \times$ ln(Openness)			0.0448	(0.1575)	0.0866	(0.1824)
Constant	0.5878	(0.4326)	1.1516*	(0.6258)	1.4789**	(0.6419)
n	868		868		868	
F-Statistics	10.9248		19.3011		126.4900	
p-value	0.0000		0.0000		0.0000	

Note: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 14: LS Estimations using $\mathbf{W} \times \ln(\text{Height})$ as dependent variable

	(D-5)		(D-6)		(D-7)	
Variables	Coef.	SE	Coef.	SE	Coef.	SE
$\mathbf{W}_{n,0}$ × Male	0.1180	(0.1872)			0.2803	(0.1927)
$\mathbf{W}_{n,0}$ × ln(Weight)	0.2625	(0.1916)			0.0643	(0.3955)
$\mathbf{W}_{n,0}$ × Siblings Help	0.2883	(0.1941)			0.1956	(0.1622)
$\mathbf{W}_{n,0}$ × Parents Help	0.1967	(0.1748)			0.2461	(0.1683)
$\mathbf{W}_{n,0}$ × Commute by Car/Taxi	0.1375	(0.1625)			0.1052	(0.1700)
$\mathbf{W}_{n,0}$ × Music	0.2489*	(0.1306)			0.0600	(0.1596)
$\mathbf{W}_{n,0}$ × ln(Cognitive)	-0.2424	(0.1828)			0.4494	(0.3957)
$\mathbf{W}_{n,0}$ × ln(Agreeableness)	-0.1306	(0.2771)			-0.1050	(0.5708)
$\mathbf{W}_{n,0}$ × ln(Conscientiousness)	0.5010**	(0.2017)			0.0665	(0.4104)
$\mathbf{W}_{n,0}$ × ln(Extraversion)	0.0614	(0.1853)			0.2481	(0.2555)
$\mathbf{W}_{n,0}$ × ln(Neuroticism)	-0.1489	(0.1897)			-0.0746	(0.3510)
$\mathbf{W}_{n,0}$ × ln(Openness)	0.1441	(0.3229)			0.3960	(0.3747)
$\mathbf{W}_{n,0}^2$ × Male			-0.4456	(0.3082)	-0.5613*	(0.3149)
$\mathbf{W}_{n,0}^2$ × ln(Weight)			0.6125	(0.5231)	0.5680	(0.6976)
$\mathbf{W}_{n,0}^2$ × Siblings Help			0.5837**	(0.2417)	0.5906**	(0.2335)
$\mathbf{W}_{n,0}^2$ × Parents Help			0.0234	(0.2432)	-0.0045	(0.2465)
$\mathbf{W}_{n,0}^2$ × Commute by Car/Taxi			-0.2487	(0.2473)	-0.2697	(0.2421)
$\mathbf{W}_{n,0}^2$ × Music			0.5045***	(0.1665)	0.4822**	(0.1975)
$\mathbf{W}_{n,0}^2$ × ln(Cognitive)			-0.5994**	(0.2816)	-1.1088*	(0.5745)
$\mathbf{W}_{n,0}^2$ × ln(Agreeableness)			0.0437	(0.4733)	0.3197	(0.8356)
$\mathbf{W}_{n,0}^2$ × ln(Conscientiousness)			1.0995***	(0.2510)	1.0402*	(0.5467)
$\mathbf{W}_{n,0}^2$ × ln(Extraversion)			-0.0054	(0.3580)	-0.2854	(0.5586)
$\mathbf{W}_{n,0}^2$ × ln(Neuroticism)			-0.1436	(0.3153)	-0.0041	(0.5480)
$\mathbf{W}_{n,0}^2$ × ln(Openness)			-0.0027	(0.5465)	-0.4294	(0.6521)
Constant	2.8956***	(0.8051)	1.8026	(1.9623)	1.3428	(1.7616)
n	868		868		868	
F-Statistics	2.8743		8.3756		267.8900	
p-value	0.0134		0.0000		0.0000	

Note: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 15: LS Estimations using $\mathbf{W} \times \ln(\text{Weight})$ as dependent variable

	(D-5)		(D-6)		(D-7)	
Variables	Coef.	SE	Coef.	SE	Coef.	SE
$\mathbf{W}_{n,0} \times$ Male	0.0836	(0.1408)			0.1835	(0.1335)
$\mathbf{W}_{n,0} \times$ ln(Height)	0.7174**	(0.3325)			1.2811	(1.4793)
$\mathbf{W}_{n,0} \times$ Siblings Help	0.1808	(0.1492)			0.1313	(0.1349)
$\mathbf{W}_{n,0} \times$ Parents Help	0.1202	(0.1302)			0.1585	(0.1339)
$\mathbf{W}_{n,0} \times$ Commute by Car/Taxi	0.0749	(0.1223)			0.0626	(0.1324)
$\mathbf{W}_{n,0} \times$ Music	0.1526	(0.0957)			0.0207	(0.1181)
$\mathbf{W}_{n,0} \times$ ln(Cognitive)	-0.1177	(0.1470)			0.2401	(0.2855)
$\mathbf{W}_{n,0} \times$ ln(Agreeableness)	0.1349	(0.2465)			-0.1997	(0.4257)
$\mathbf{W}_{n,0} \times$ ln(Conscientiousness)	0.5276***	(0.1917)			-0.0289	(0.3189)
$\mathbf{W}_{n,0} \times$ ln(Extraversion)	0.1552	(0.1399)			0.1207	(0.2145)
$\mathbf{W}_{n,0} \times$ ln(Neuroticism)	0.0531	(0.1750)			-0.1283	(0.2683)
$\mathbf{W}_{n,0} \times$ ln(Openness)	0.2478	(0.2398)			0.3701	(0.2943)
$\mathbf{W}^2_{n,0} \times$ Male			-0.2662	(0.2209)	-0.3274	(0.2211)
$\mathbf{W}^2_{n,0} \times$ ln(Height)			1.1151**	(0.5044)	-0.2210	(1.6553)
$\mathbf{W}^2_{n,0} \times$ Siblings Help			0.3890**	(0.1887)	0.3742**	(0.1821)
$\mathbf{W}^2_{n,0} \times$ Parents Help			-0.0328	(0.1957)	-0.0597	(0.1952)
$\mathbf{W}^2_{n,0} \times$ Commute by Car/Taxi			-0.2155	(0.1877)	-0.2338	(0.1884)
$\mathbf{W}^2_{n,0} \times$ Music			0.3344**	(0.1344)	0.3274**	(0.1590)
$\mathbf{W}^2_{n,0} \times$ ln(Cognitive)			-0.3279	(0.2107)	-0.6054	(0.4380)
$\mathbf{W}^2_{n,0} \times$ ln(Agreeableness)			0.3474	(0.4004)	0.6676	(0.6866)
$\mathbf{W}^2_{n,0} \times$ ln(Conscientiousness)			1.0425***	(0.2321)	1.0599**	(0.4446)
$\mathbf{W}^2_{n,0} \times$ ln(Extraversion)			0.1596	(0.2918)	0.0006	(0.4765)
$\mathbf{W}^2_{n,0} \times$ ln(Neuroticism)			0.1233	(0.2948)	0.2734	(0.4803)
$\mathbf{W}^2_{n,0} \times$ ln(Openness)			0.1558	(0.4027)	-0.2917	(0.4938)
Constant	-0.5989	(1.6537)	-2.4319	(2.4581)	-2.3797	(2.4980)
n	868		868		868	
F-Statistics	3.2110		7.3745		28.3500	
p-value	0.0072		0.0000		0.0000	

Note: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 16: LS Estimations using $\mathbf{W} \times$ Siblings Help as dependent variable

	(D-5)		(D-6)		(D-7)	
Variables	Coef.	SE	Coef.	SE	Coef.	SE
$\mathbf{W}_{n,0} \times$ Male	-0.0436	(0.0379)			-0.0275	(0.0345)
$\mathbf{W}_{n,0} \times$ ln(Height)	-0.0999	(0.0919)			-0.6092	(0.3855)
$\mathbf{W}_{n,0} \times$ ln(Weight)	0.0392	(0.0863)			0.1094	(0.0930)
$\mathbf{W}_{n,0} \times$ Parents Help	0.0451	(0.0332)			0.0330	(0.0301)
$\mathbf{W}_{n,0} \times$ Commute by Car/Taxi	0.0639*	(0.0326)			0.0656*	(0.0372)
$\mathbf{W}_{n,0} \times$ Music	0.0158	(0.0352)			-0.0173	(0.0335)
$\mathbf{W}_{n,0} \times$ ln(Cognitive)	-0.0248	(0.0345)			-0.0716	(0.0642)
$\mathbf{W}_{n,0} \times$ ln(Agreeableness)	-0.0670	(0.0598)			-0.0254	(0.0959)
$\mathbf{W}_{n,0} \times$ ln(Conscientiousness)	0.0854**	(0.0399)			0.0177	(0.0724)
$\mathbf{W}_{n,0} \times$ ln(Extraversion)	-0.0416	(0.0547)			0.0135	(0.0898)
$\mathbf{W}_{n,0} \times$ ln(Neuroticism)	-0.0215	(0.0378)			-0.0045	(0.0657)
$\mathbf{W}_{n,0} \times$ ln(Openness)	-0.0335	(0.0614)			-0.0775	(0.0799)
$\mathbf{W}^2_{n,0} \times$ Male			-0.1188**	(0.0597)	-0.1127*	(0.0644)
$\mathbf{W}^2_{n,0} \times$ ln(Height)			0.0725	(0.1319)	0.6017	(0.4002)
$\mathbf{W}^2_{n,0} \times$ ln(Weight)			-0.1291	(0.0842)	-0.1695*	(0.1014)
$\mathbf{W}^2_{n,0} \times$ Parents Help			0.0610	(0.0602)	0.0652	(0.0592)
$\mathbf{W}^2_{n,0} \times$ Commute by Car/Taxi			0.0416	(0.0510)	0.0415	(0.0519)
$\mathbf{W}^2_{n,0} \times$ Music			0.0947**	(0.0417)	0.1039***	(0.0402)
$\mathbf{W}^2_{n,0} \times$ ln(Cognitive)			0.0496	(0.0587)	0.1322	(0.1107)
$\mathbf{W}^2_{n,0} \times$ ln(Agreeableness)			-0.1213	(0.1071)	-0.1176	(0.1763)
$\mathbf{W}^2_{n,0} \times$ ln(Conscientiousness)			0.1726***	(0.0634)	0.1475	(0.1139)
$\mathbf{W}^2_{n,0} \times$ ln(Extraversion)			-0.0354	(0.0873)	-0.0551	(0.1422)
$\mathbf{W}^2_{n,0} \times$ ln(Neuroticism)			-0.0623	(0.0651)	-0.0665	(0.1185)
$\mathbf{W}^2_{n,0} \times$ ln(Openness)			-0.0080	(0.1083)	0.0776	(0.1576)
Constant	0.7144**	(0.3385)	0.4611	(0.5473)	0.5524	(0.6007)
n	868		868		868	
F-Statistics	1.7506		4.3328		10.4200	
p-value	0.1174		0.0011		0.0000	

Note: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 17: LS Estimations using $\mathbf{W} \times$ Parents Help as dependent variable

	(D-5)		(D-6)		(D-7)	
Variables	Coef.	SE	Coef.	SE	Coef.	SE
$\mathbf{W}_{n,0} \times$ Male	0.0252	(0.0534)			0.0333	(0.0564)
$\mathbf{W}_{n,0} \times$ ln(Height)	0.2073	(0.1268)			0.5553	(0.4865)
$\mathbf{W}_{n,0} \times$ ln(Weight)	-0.1920**	(0.0948)			-0.1684*	(0.0909)
$\mathbf{W}_{n,0} \times$ Siblings Help	0.0617	(0.0435)			0.0520	(0.0438)
$\mathbf{W}_{n,0} \times$ Commute by Car/Taxi	0.0236	(0.0373)			0.0162	(0.0412)
$\mathbf{W}_{n,0} \times$ Music	0.0308	(0.0399)			-0.0005	(0.0379)
$\mathbf{W}_{n,0} \times$ ln(Cognitive)	0.0143	(0.0390)			0.0399	(0.0814)
$\mathbf{W}_{n,0} \times$ ln(Agreeableness)	-0.0304	(0.0722)			-0.0502	(0.1190)
$\mathbf{W}_{n,0} \times$ ln(Conscientiousness)	0.1347**	(0.0646)			0.0392	(0.0911)
$\mathbf{W}_{n,0} \times$ ln(Extraversion)	0.0377	(0.0427)			0.1117	(0.0750)
$\mathbf{W}_{n,0} \times$ ln(Neuroticism)	-0.0500	(0.0554)			-0.1029	(0.0751)
$\mathbf{W}_{n,0} \times$ ln(Openness)	0.0034	(0.0716)			-0.0387	(0.0908)
$\mathbf{W}^2_{n,0} \times$ Male			0.0146	(0.0651)	-0.0097	(0.0735)
$\mathbf{W}^2_{n,0} \times$ ln(Height)			0.0980	(0.1905)	-0.3670	(0.5233)
$\mathbf{W}^2_{n,0} \times$ ln(Weight)			-0.0499	(0.1486)	0.0217	(0.1424)
$\mathbf{W}^2_{n,0} \times$ Siblings Help			0.1268***	(0.0433)	0.0972**	(0.0477)
$\mathbf{W}^2_{n,0} \times$ Commute by Car/Taxi			0.0172	(0.0634)	0.0378	(0.0602)
$\mathbf{W}^2_{n,0} \times$ Music			0.0976**	(0.0454)	0.0989**	(0.0454)
$\mathbf{W}^2_{n,0} \times$ ln(Cognitive)			-0.0012	(0.0618)	-0.0401	(0.1282)
$\mathbf{W}^2_{n,0} \times$ ln(Agreeableness)			-0.0425	(0.1254)	0.0448	(0.1980)
$\mathbf{W}^2_{n,0} \times$ ln(Conscientiousness)			0.2338**	(0.1002)	0.1852	(0.1538)
$\mathbf{W}^2_{n,0} \times$ ln(Extraversion)			-0.0281	(0.0860)	-0.1573	(0.1540)
$\mathbf{W}^2_{n,0} \times$ ln(Neuroticism)			-0.0400	(0.0898)	0.0910	(0.1330)
$\mathbf{W}^2_{n,0} \times$ ln(Openness)			0.0027	(0.1347)	0.0405	(0.1839)
Constant	0.2101	(0.4627)	0.1457	(0.6395)	0.0231	(0.6233)
n	868		868		868	
F-Statistics	3.0899		1.8041		25.4400	
p-value	0.0090		0.1057		0.0000	

Note: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 18: LS Estimations using $\mathbf{W} \times$ Commute by Car/Taxi as dependent variable

	(D-5)		(D-6)		(D-7)	
Variables	Coef.	SE	Coef.	SE	Coef.	SE
$\mathbf{W}_{n,0} \times$ Male	0.0167	(0.0482)			0.0494	(0.0512)
$\mathbf{W}_{n,0} \times$ ln(Height)	0.1140	(0.1299)			-0.1766	(0.3776)
$\mathbf{W}_{n,0} \times$ ln(Weight)	0.0206	(0.0859)			0.1109	(0.0949)
$\mathbf{W}_{n,0} \times$ Siblings Help	0.0464	(0.0444)			0.0344	(0.0441)
$\mathbf{W}_{n,0} \times$ Parents Help	0.0028	(0.0290)			0.0043	(0.0314)
$\mathbf{W}_{n,0} \times$ Music	0.0211	(0.0360)			-0.0033	(0.0404)
$\mathbf{W}_{n,0} \times$ ln(Cognitive)	-0.0589	(0.0596)			0.0425	(0.0971)
$\mathbf{W}_{n,0} \times$ ln(Agreeableness)	0.0527	(0.0563)			-0.0520	(0.1007)
$\mathbf{W}_{n,0} \times$ ln(Conscientiousness)	0.1208***	(0.0422)			-0.0152	(0.0892)
$\mathbf{W}_{n,0} \times$ ln(Extraversion)	0.0581	(0.0519)			-0.0253	(0.0891)
$\mathbf{W}_{n,0} \times$ ln(Neuroticism)	0.0323	(0.0419)			-0.0002	(0.0748)
$\mathbf{W}_{n,0} \times$ ln(Openness)	-0.0344	(0.0811)			0.0570	(0.1415)
$\mathbf{W}^2_{n,0} \times$ Male			-0.1128*	(0.0684)	-0.1296**	(0.0587)
$\mathbf{W}^2_{n,0} \times$ ln(Height)			0.3602*	(0.2035)	0.4488	(0.4245)
$\mathbf{W}^2_{n,0} \times$ ln(Weight)			-0.1240	(0.1436)	-0.1462	(0.1429)
$\mathbf{W}^2_{n,0} \times$ Siblings Help			0.1113***	(0.0398)	0.1211***	(0.0415)
$\mathbf{W}^2_{n,0} \times$ Parents Help			0.0816	(0.0527)	0.0838	(0.0510)
$\mathbf{W}^2_{n,0} \times$ Music			0.0342	(0.0376)	0.0350	(0.0381)
$\mathbf{W}^2_{n,0} \times$ ln(Cognitive)			-0.0991	(0.0765)	-0.1547	(0.1215)
$\mathbf{W}^2_{n,0} \times$ ln(Agreeableness)			0.1499	(0.1091)	0.2124	(0.1820)
$\mathbf{W}^2_{n,0} \times$ ln(Conscientiousness)			0.2543***	(0.0743)	0.2695*	(0.1472)
$\mathbf{W}^2_{n,0} \times$ ln(Extraversion)			0.1712***	(0.0663)	0.1959*	(0.1155)
$\mathbf{W}^2_{n,0} \times$ ln(Neuroticism)			0.0603	(0.0730)	0.0494	(0.1346)
$\mathbf{W}^2_{n,0} \times$ ln(Openness)			-0.1452	(0.0956)	-0.2129	(0.1814)
Constant	-0.1795	(0.4529)	-0.9086	(0.6702)	-0.8397	(0.6981)
n	868		868		868	
F-Statistics	2.8197		6.3966		21.4400	
p-value	0.0148		0.0001		0.0000	

Note: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 19: LS Estimations using $\mathbf{W} \times$ Music as dependent variable

	(D-5)		(D-6)		(D-7)	
Variables	Coef.	SE	Coef.	SE	Coef.	SE
$\mathbf{W}_{n,0} \times$ Male	-0.0666	(0.0557)			-0.0043	(0.0557)
$\mathbf{W}_{n,0} \times$ ln(Height)	0.3127**	(0.1515)			-0.1204	(0.5234)
$\mathbf{W}_{n,0} \times$ ln(Weight)	-0.3884***	(0.1083)			-0.2920***	(0.1108)
$\mathbf{W}_{n,0} \times$ Siblings Help	0.0005	(0.0389)			-0.0089	(0.0356)
$\mathbf{W}_{n,0} \times$ Parents Help	0.0761	(0.0708)			0.0791	(0.0626)
$\mathbf{W}_{n,0} \times$ Commute by Car/Taxi	0.0057	(0.0499)			-0.0038	(0.0485)
$\mathbf{W}_{n,0} \times$ ln(Cognitive)	-0.1652***	(0.0637)			-0.0564	(0.0934)
$\mathbf{W}_{n,0} \times$ ln(Agreeableness)	0.0980	(0.0797)			0.0883	(0.1335)
$\mathbf{W}_{n,0} \times$ ln(Conscientiousness)	0.0190	(0.0478)			-0.0632	(0.0964)
$\mathbf{W}_{n,0} \times$ ln(Extraversion)	-0.0369	(0.0520)			-0.1098	(0.0681)
$\mathbf{W}_{n,0} \times$ ln(Neuroticism)	0.0477	(0.0569)			-0.0444	(0.0774)
$\mathbf{W}_{n,0} \times$ ln(Openness)	-0.0273	(0.0824)			-0.0120	(0.1017)
$\mathbf{W}^2_{n,0} \times$ Male			-0.2865***	(0.0669)	-0.2900***	(0.0804)
$\mathbf{W}^2_{n,0} \times$ ln(Height)			0.3415	(0.2258)	0.6147	(0.5420)
$\mathbf{W}^2_{n,0} \times$ ln(Weight)			-0.2853*	(0.1620)	-0.1608	(0.2271)
$\mathbf{W}^2_{n,0} \times$ Siblings Help			0.1136*	(0.0687)	0.0990**	(0.0445)
$\mathbf{W}^2_{n,0} \times$ Parents Help			0.0645	(0.0680)	0.0614	(0.0753)
$\mathbf{W}^2_{n,0} \times$ Commute by Car/Taxi			-0.0844	(0.0652)	-0.0823	(0.0755)
$\mathbf{W}^2_{n,0} \times$ ln(Cognitive)			-0.2039**	(0.0905)	-0.1334	(0.1251)
$\mathbf{W}^2_{n,0} \times$ ln(Agreeableness)			0.0838	(0.1486)	-0.0304	(0.2709)
$\mathbf{W}^2_{n,0} \times$ ln(Conscientiousness)			0.0887	(0.1102)	0.1801	(0.1672)
$\mathbf{W}^2_{n,0} \times$ ln(Extraversion)			0.0247	(0.1097)	0.1649	(0.1593)
$\mathbf{W}^2_{n,0} \times$ ln(Neuroticism)			0.0749	(0.0914)	0.1379	(0.1367)
$\mathbf{W}^2_{n,0} \times$ ln(Openness)			-0.0119	(0.1453)	-0.0084	(0.2091)
Constant	0.5647	(0.5552)	0.1232	(0.8391)	-0.0420	(0.8874)
n	868		868		868	
F-Statistics	3.7731		3.65		13.4900	
p-value	0.0027		0.0000		0.0000	

Note: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 20: LS Estimations using $\mathbf{W}y$ as dependent variable

	(D-8)		(D-9)		(D-10)	
Variables	Coef.	SE	Coef.	SE	Coef.	SE
$\mathbf{W}_{n,*}$ × Male	-0.0504	(0.0487)			-0.0496	(0.0413)
$\mathbf{W}_{n,*}$ × ln(Height)	0.8742***	(0.2639)			0.3636**	(0.1728)
$\mathbf{W}_{n,*}$ × ln(Weight)	-0.1477	(0.1316)			0.0612	(0.0988)
$\mathbf{W}_{n,*}$ × Siblings Help	0.0403	(0.0444)			0.0544	(0.0394)
$\mathbf{W}_{n,*}$ × Parents Help	0.0065	(0.0348)			-0.0557	(0.0400)
$\mathbf{W}_{n,*}$ × Commute Car/Taxi	0.0122	(0.0539)			-0.0089	(0.0474)
$\mathbf{W}_{*}$ × Music	0.0651	(0.0436)			-0.0012	(0.0365)
$\mathbf{W}_{n,*}$ × ln(Cognitive)	-0.2676**	(0.1081)			-0.2059***	(0.0729)
$\mathbf{W}_{n,*}$ × ln(Agreeableness)	-0.0154	(0.2255)			-0.2280*	(0.1287)
$\mathbf{W}_{n,*}$ × ln(Conscientious)	0.3370**	(0.1402)			0.1333*	(0.0731)
$\mathbf{W}_{n,*}$ × ln(Extraversion)	0.0306	(0.1665)			0.1601*	(0.0826)
$\mathbf{W}_{n,*}$ × ln(Neuroticism)	-0.0098	(0.1254)			0.0122	(0.0625)
$\mathbf{W}_{n,*}$ × ln(Openness)	0.2343	(0.2068)			-0.0649	(0.1187)
$\mathbf{W}^2_{n,*}$ × Male			-0.2182	(0.1743)	-0.0939	(0.1271)
$\mathbf{W}^2_{n,*}$ × ln(Height)			0.9572*	(0.4962)	0.5604	(0.4129)
$\mathbf{W}^2_{n,*}$ × ln(Weight)			-0.0553	(0.4378)	-0.1974	(0.2979)
$\mathbf{W}^2_{n,*}$ × Siblings Help			0.2420	(0.2819)	0.2458	(0.1603)
$\mathbf{W}^2_{n,*}$ × Parents Help			0.0493	(0.1802)	0.0326	(0.0824)
$\mathbf{W}^2_{n,*}$ × Commute Car/Taxi			0.5050***	(0.1750)	0.0522	(0.1192)
$\mathbf{W}^2_{n,*}$ × Music			-0.1076	(0.0977)	0.1880***	(0.0484)
$\mathbf{W}^2_{n,*}$ × ln(Cognitive)			-0.3588**	(0.1464)	-0.0596	(0.1343)
$\mathbf{W}^2_{n,*}$ × ln(Agreeableness)			0.0570	(0.2684)	0.2458	(0.2486)
$\mathbf{W}^2_{n,*}$ × ln(Conscientious)			0.4525**	(0.2023)	0.2621*	(0.1535)
$\mathbf{W}^2_{n,*}$ × ln(Extraversion)			-0.0010	(0.1893)	-0.1465	(0.1558)
$\mathbf{W}^2_{n,*}$ × ln(Neuroticism)			0.0569	(0.1904)	-0.0245	(0.1524)
$\mathbf{W}^2_{n,*}$ × ln(Openness)			0.4369	(0.2845)	0.3608*	(0.1930)
Constant	0.2689	(1.1172)	-0.3995	(1.6556)	-0.1877	(1.4016)
n	868		868		868	
F-Statistics	20.2968		22.4297		40.1600	
p-value	0.0000		0.0000		0.0000	

Note: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

References

Jenish, Nazgul and Ingmar R Prucha. 2009. "Central limit theorems and uniform laws of large numbers for arrays of random fields." *Journal of econometrics* 150 (1):86–98.

Kojevnikov, Denis, Vadim Marmer, and Kyungchul Song. 2021. "Limit Theorems for Network Dependent Random Variables." *Journal of Econometrics* 222 (2):882–908.